*The Journal of*

# THE ROYAL AERONAUTICAL SOCIETY

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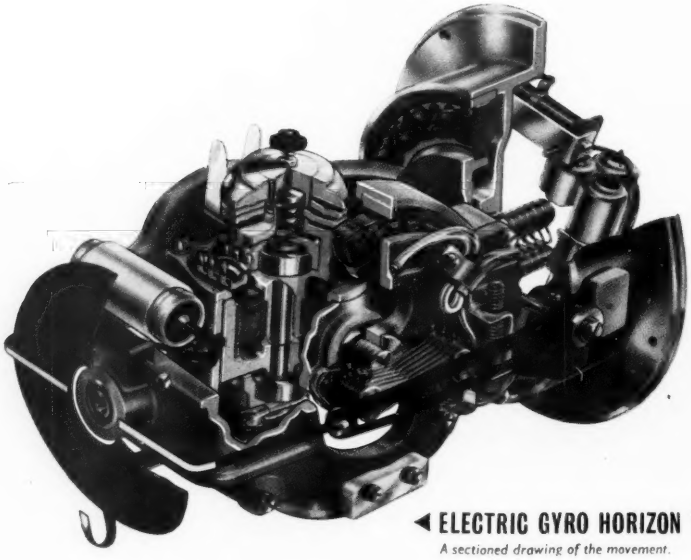
# SPERRY

## *GYROSCOPIC INSTRUMENTS FOR NAVIGATION AND PRECISE CONTROL OF AIRCRAFT*



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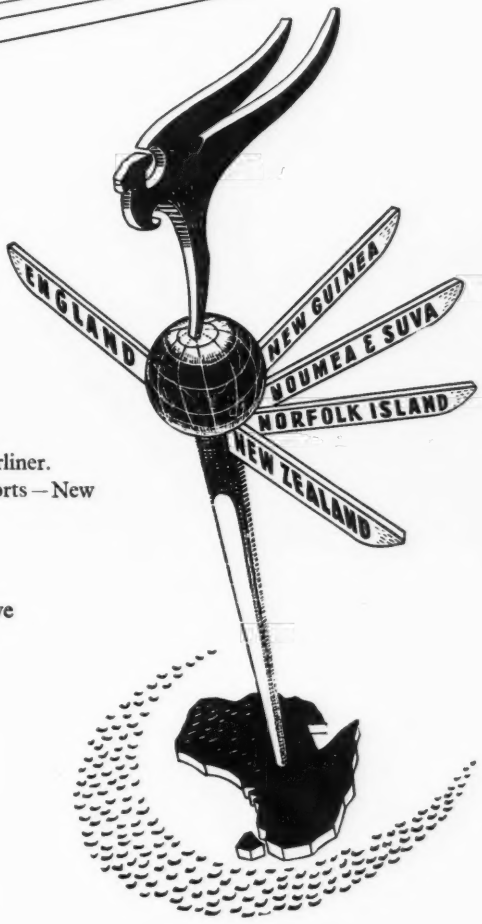


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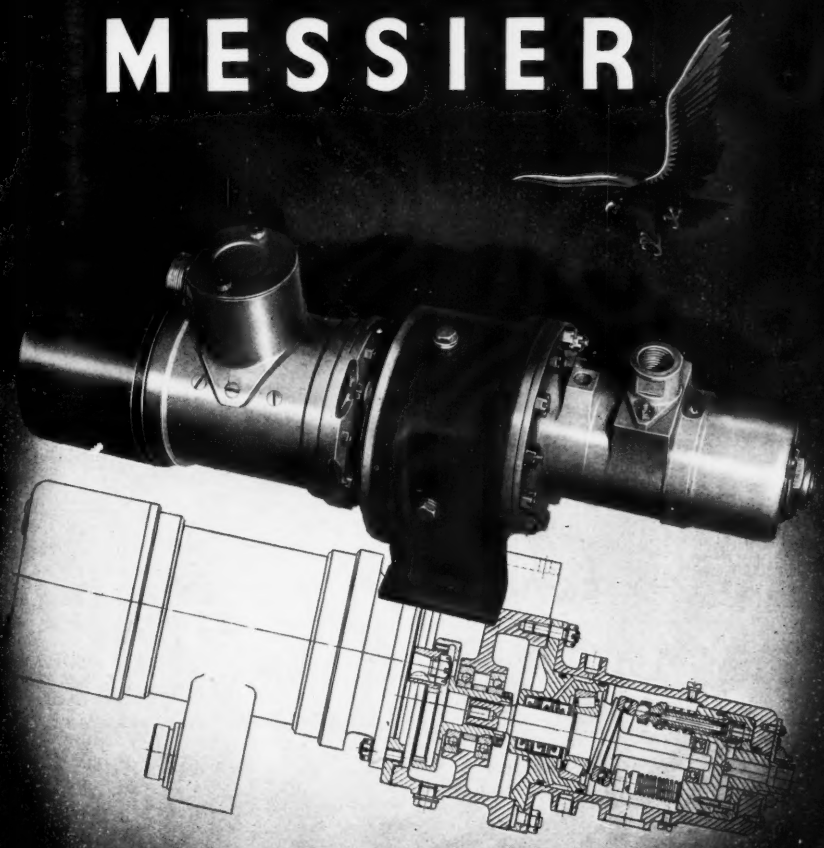

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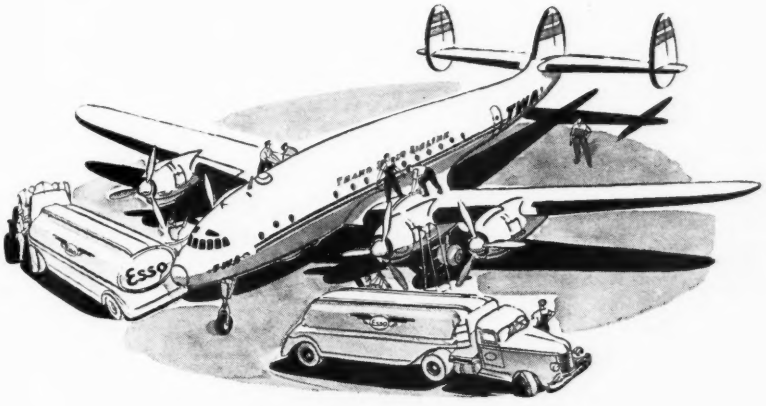


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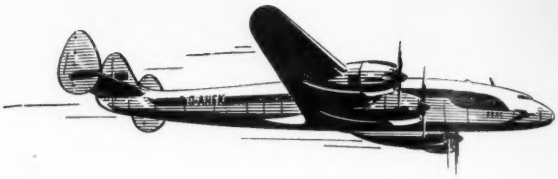

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THE ROYAL AERONAUTICAL SOCIETY

# ACTIVITIES of the GRADUATE & STUDENT SECTION

*May* 1949

## ANNUAL GENERAL MEETING

THE Thirteenth Annual General Meeting of the Section was held at the Society at 7.30 p.m. on Wednesday 23rd March 1949. A summary of the proceedings is given below.

1. The minutes of the previous Annual General Meeting were read and confirmed. It was asked whether any action had been taken over the motion, passed at the previous meeting, that "this meeting considers it highly desirable that the Section should have two representatives on Council, one to be the Section Chairman, and the other to be elected from the Section at the Annual General Meeting." It was explained that the motion had been included in the report on the Twelfth Annual General Meeting which was sent to the President and Secretary.
2. The Hon. Secretary's Annual Report on the Section's activities during the Session 1948-49 was read and adopted.
3. The Committee for the Session 1949-50 was elected. A list of the members elected is given below.
4. A number of suggestions were made for lectures, visits and film shows to be held during the ensuing session.
5. It was agreed that efforts should be made to hold the Conversazione, which had had to be abandoned last session, in October 1949.
6. A vote of thanks to Captain Pritchard and the staff of the Society for their help and advice during the year was carried unanimously.

## COMMITTEE

The Committee for the year 1949-50 consists of the following members:—

*Chairman*: J. F. W. HOUSEGO, *Graduate* (Handley Page Ltd.).
*Vice-Chairman*: P. T. FINK, *Graduate* (Imperial College).
*Hon. Secretary*: M. C. CAMPION, *Graduate* (Vickers-Armstrongs Ltd.).
*Assistant Hon. Secretary*: D. A. THURGOOD, *Student* (Vickers-Armstrongs Ltd.).
*Committee*: M. K. BOWDEN, *Student* (Northampton Engineering College).
W. M. BRADY, *Graduate* (Vickers-Armstrongs Ltd.).
J. C. GIBBINGS, *Graduate* (Queen Mary College).
F. G. IRVING, *Graduate* (Imperial College).
J. R. MILLER, *Graduate* (Hawker Aircraft Ltd.).
J. K. PUDNEY, *Student* (Fairey Aviation Co. Ltd.).
J. G. ROXBURGH, *Graduate* (Handley Page (Reading) Ltd.).
A. STANBROOK, *Student* (Fairey Aviation Co. Ltd.).

## JOINT MEETING ON "WIND TUNNELS"

A joint meeting of the Section, of the London Graduates' Section of the Institution of Mechanical Engineers, and of the London Students' Section of the Institution of Electrical Engineers, has been provisionally arranged for 1950, to discuss "Wind Tunnels." It is proposed to have one short paper (lasting about twenty minutes) by a member of each body, on an appropriate aspect of the subject, and to follow these with an open discussion.

Members of the Section who are willing to represent the Society at this meeting should write to the Hon. Secretary, mentioning the aspect (preferably not highly specialised) on which they are prepared to speak.

## LECTURES

Lectures given by members of the Section should be a prominent feature of the Section's programme. In the past few years, few papers have been read by Graduates or Students.

During this Session, it is hoped that at least half the lectures will be given by Graduates and Students At present, one member of the Section has offered to deliver a paper, which means that at least four more are required. The Hon. Secretary will be pleased to hear from any member who is willing to give a lecture.

This can be either a full length paper, occupying a full evening meeting (*i.e.* about 1¼ hours), or a short (20 minute) paper, in which case the meeting would consist of three such papers.

THE ROYAL AERONAUTICAL SOCIETY

# PAPERS FOR THE JOURNAL

The Council have set aside an annual sum of £250 for the award of premiums for papers published in the JOURNAL. Both members and non-members are invited to contribute original papers on their own special subjects. The following notes on the preparation of papers for the JOURNAL are given to assist contributors:—

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The copyright of every paper printed in the JOURNAL shall be the property of Aerial Science Limited. If the author makes use of copyright material in his paper or information obtained by reason of his employment or otherwise, he must state clearly in his covering letter, or in the paper, that consent has been given for the use of such material.

**MANUSCRIPTS**

Papers must be in English, in the third person, and typed on one side of the paper only, with double spacing and wide margins. When submitted they must be in their *final form for publication.* Only typographical errors may be corrected in proofs. Titles of papers should be brief.

Where practicable, a summary of not more than 250 words should be given at the beginning of the paper, giving its scope and conclusions.

All references should be numbered in the text where they occur, and the complete list of references given at the end of the manuscript. These references will be published at the end of the text. Footnotes should be numbered consecutively. Tables should be kept as concise as possible.

**ILLUSTRATIONS**

Illustrations must be drawn so that they will reduce to column or two-column width, that is to $2\frac{5}{8}$ or $5\frac{1}{2}$ inches. Full page illustrations must reduce to $5\frac{1}{2}$ inches by 8 inches. All drawings must be in black ink on white paper or tracing cloth.

Lettering and figures on drawings must be in pencil only. Drawings should be posted flat or rolled.

Photographs should not be less than half plate in size and must be clear black and white glossy prints.

Every drawing and photograph should have on the back its figure number and title.

**MATHEMATICS**

Only very simple symbols and formulæ should be typewritten. All others should be written carefully by hand in ink. Ample space for marking should be allowed above and below all equations. Greek letters should be designated by name in the margin.

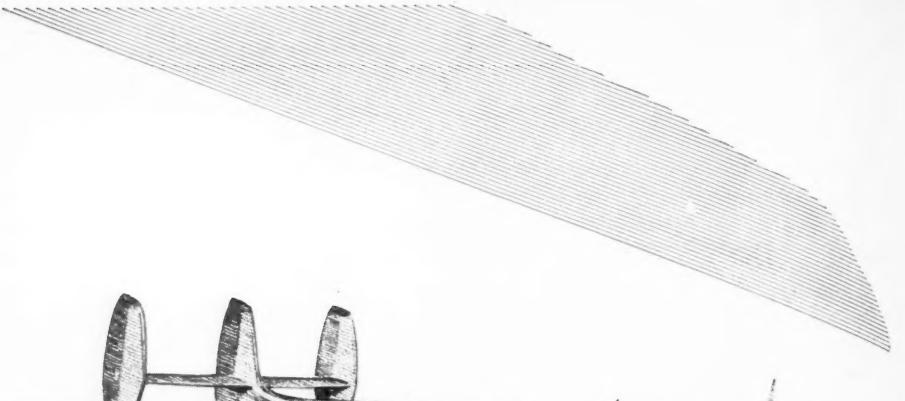
The difference between capital and lower-case letters should be clearly shown; care should be taken to avoid confusion between zero (0) and the letter o, between the numeral one (1) and the letter l, between alpha and a, kappa and k, mu and u, nu and v, eta and n.

All subscripts and exponents should be clearly marked, and dots, bars, and so on, over letters should be avoided as far as possible.

Square roots of complicated expressions should be written with the exponent $\frac{1}{2}$ rather than with the sign $\surd$.

Complicated exponents and subscripts should be avoided. Any complicated expression that recurs frequently should be represented by a special symbol.

Unless an abbreviation is one recognised as standard practice, both in this and other English-speaking countries, for example, b.h.p. for brake horse power, the meaning should be given in full on its first use.



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*The Society offers a number of valuable awards, most of them annually. Full particulars of the conditions attaching to these awards may be obtained on application to the Secretary.*

**Society's Gold Medal**
Conferred for work of an outstanding or fundamental nature in aeronautics.

**British Gold Medal for Aeronautics**
Awarded for an achievement leading to advancement in aeronautical science.

**Simms Gold Medal**
Awarded for the best paper read in any year before the Society on any science allied to aeronautics, *e.g.*, meteorology, wireless telegraphy, instruments.

**The George Taylor (of Australia) Gold Medal**
Awarded for the most valuable paper submitted or read during the previous session.

**Wakefield Gold Medal**
Awarded to the designer of any invention or apparatus tending towards safety in flying; open to members or non-members.

**Society's Silver Medal**
Awarded for some advance in aeronautical design.

**British Silver Medal for Aeronautics**
Awarded for an achievement leading to advancement in aeronautical science.

**Society's Bronze Medal**
Awarded under the same conditions as those for the Silver Medal, but for some less important advance in aeronautical design.

**Wilbur Wright Memorial Premium**
The Wilbur Wright Memorial Lecture is held annually, a premium of £75 being awarded to the lecturer invited by the Council to deliver the lecture. The lecture is usually given alternately by an American and an Englishman, and is the most important aeronautical lecture of the year.

**British Commonwealth and Empire Lecture**
The British Commonwealth and Empire Lecture is delivered annually by a lecturer chosen in alternate years from the British Dominions and Colonies and Great Britain.
The British Commonwealth and Empire Lecture has a premium of £50 attached to it, and in the case of lecturers coming from the Dominions and Colonies an allowance will be paid towards the Lecturer's expenses.

**R.38 Memorial Prize**
Offered annually for the best paper received by the Society on some subject of a technical nature in the science of aeronautics, preference being given to papers which relate to airships. The prize is twenty-five guineas.

**The Herbert Akroyd Stuart Lectures**
Under the will of the late Mr. Herbert Akroyd Stuart a sum of £700 is held in trust by the Society for the offer of a prize every two years, for the best paper or lecture read or given before the Society dealing with the Origin and Development of Heavy-oil Aero-Engines. The prize is open to members and non-members.

**Edward Busk Memorial Prize**
Offered annually for the best paper received by the Society on some subect of a technical nature in connection with aeroplanes (including seaplanes). Its value is twenty guineas.

**Pilcher Memorial Prize**
Offered annually for the best paper by a Student on heavier-than-air craft or any analogous subject. Its value is five guineas.

**Usborne Prize**
Offered annually for the best paper by a Student on some subject in connection with Aero-Engines. Its value is five pounds.

**Major Baden-Powell Memorial Prize**
Awarded in May and December of each year to the Student who is considered the best student by the examiners in the Society's Associate Fellowship examinations. Its value is three guineas.

**Elliott Memorial Prize**
Awarded twice yearly to the apprentice at Halton who has the highest percentage of marks in the passing-out examination. Its value is two and a half guineas for each award.

**R. P. Alston Memorial Prize**
Awarded to any Graduate or Student of the Society for work done leading to improvement in the safety of aircraft, and particularly for improvement in stability and control. Its value is approximately five pounds.

**Branch Prize**
The Council offers an annual prize of twenty guineas for the best paper read before the Branches during the previous lecture Session. The prize is open to any member of the Society or of any Branch.

**Journal Premium Awards**
The Council has set aside an annual sum of £250 to be given as premium awards to authors of papers published in the JOURNAL.

AT54

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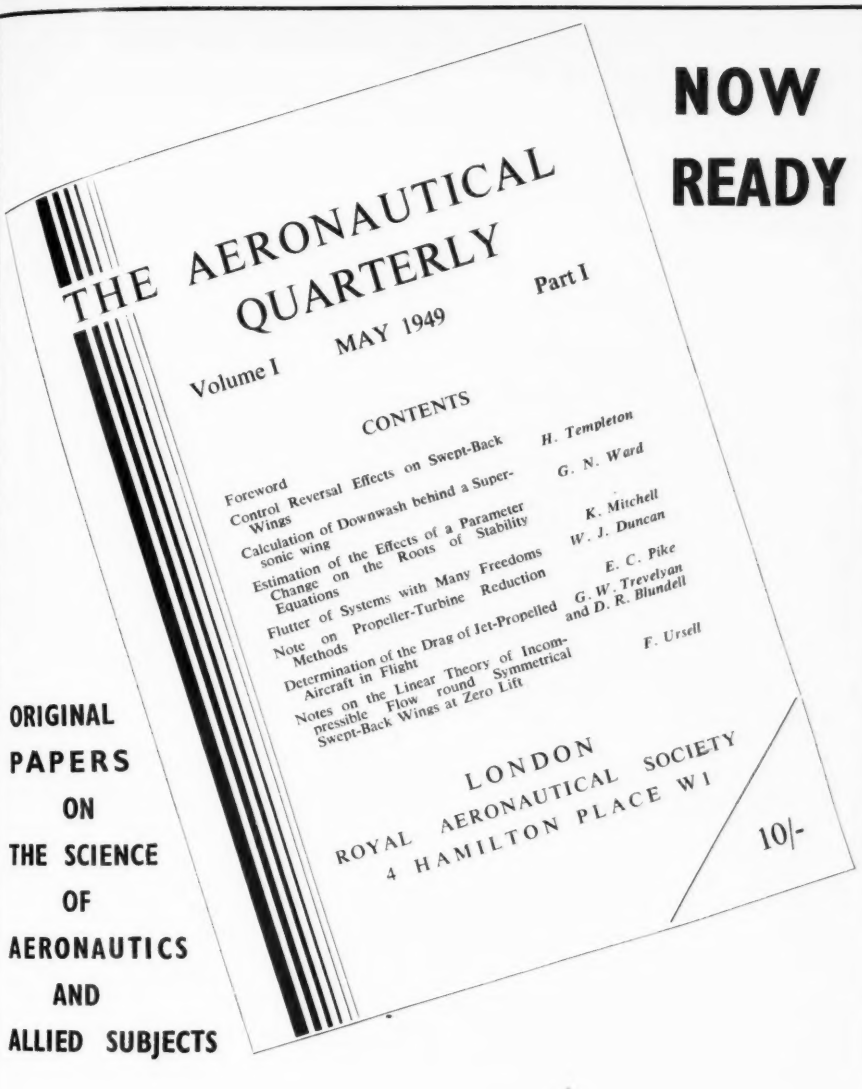
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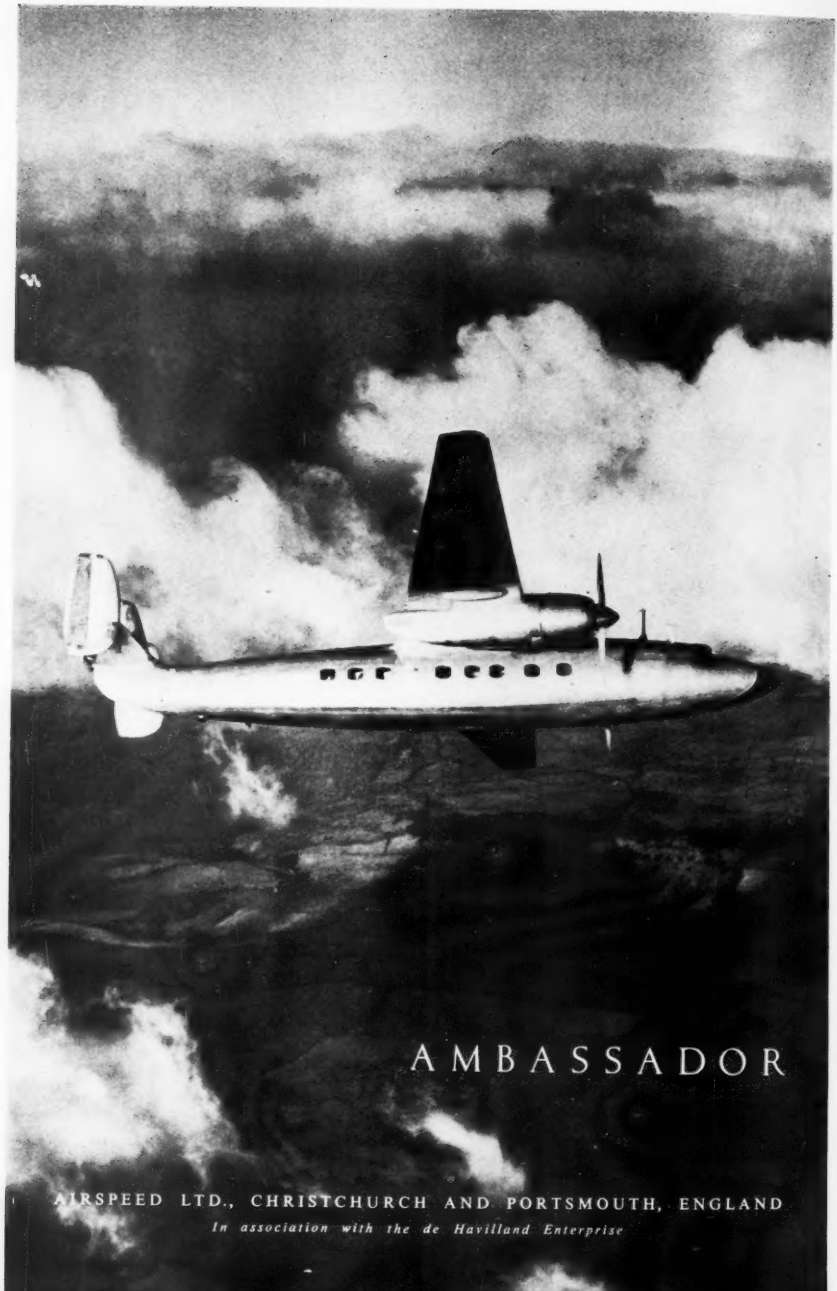
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# THE MODERN AUTO-PILOT

by

F. W. MEREDITH, B.A., F.R.Ae.S.

The 764th Lecture was read before the Royal Aeronautical Society on Thursday 13th January 1949, at the Institution of Civil Engineers, Great George Street, London, S.W.1. Dr. H. Roxbee Cox, B.Sc., D.I.C., F.R.Ae.S., F.I.Ae.S., President of the Society, introduced the Lecturer, Mr. F. W. Meredith, B.A., F.R.Ae.S., now in charge of research on automatic pilots and gyro instruments with S. Smith and Sons (England) Ltd.

## RETROSPECT

WHEN in 1937, I last had the privilege of lecturing to the Royal Aeronautical Society on the subject, the automatic pilot was still in its infancy. The automatic pilot then described was known as the Mark IA. It may now be revealed that it had developed out of researches at the Royal Aircraft Establishment in the field that came to be known during the Second World War as Special Weapons. The Royal Navy had been backing the work, primarily with the object of developing a pilotless target aeroplane.

Under the guidance of R. McKinnon Wood, at that time in charge of the Aerodynamics Department of the R.A.E., the World's first successful pilotless aeroplane was developed as an anti-aircraft target in 1925, after a series of abortive attempts in which failure was due primarily to instability arising from the unsuspected coupling between the yawing and rolling motions caused by downward tilt, in relation to the flight path, of the rotor axis of the azimuth gyroscope used to control the rudder. An aneroid control was provided to govern the transition from a climb with fixed elevators to level flight at an altitude of 2,000 ft. and subsequently, to control the elevator angle in accordance with height deviation. The rotor axis of the azimuth gyroscope was arranged to be horizontal in the plane of symmetry with a view to a maximum latitude for precession during the level flight condition but this arrangement provided the downward tilt, to which I have referred, during the climb. What had been overlooked was that this downward tilt caused motion of the outer gimbal ring or "false yaw" when the aeroplane rolled about the flight path. Thus the rudder moved in response to roll as well as yaw and the result was that the long period roll and yaw oscillation was negatively damped.

It was my good fortune that after Mr. McKinnon Wood enlisted my aid on this work, and while watching cinema records of the divergent oscillations which developed when the aeroplane left the catapult on a destroyer, I discovered the explanation of the phenomenon. I say good fortune, because this discovery probably resulted in my devoting most of my working life in the intensely interesting field of auto-pilot development.

Shortly after the success of the first pilotless target aeroplane, the first flying bomb was produced under the code name "Larynx," a name chosen to avoid repeated reference to the term Long Range Gun and to identify the pilotless aeroplane fitted with a Lynx engine. Fig. 1 shows a photograph taken in September 1928 of the "Larynx" on the catapult of the launching destroyer. It was designed to carry a 250 lb. bomb at a speed of 200 m.p.h. up to a range of 200 miles.

Although radio control was used in the target aeroplane and a transmitter was used in the development stages of the "Larynx" for the purpose of keeping track of the flight and its termination, it was intended that the weapon should be entirely under mechanical control to avoid the possibility of defensive radio methods. Incidentally, this aeroplane was the first application of the principle of combining the short term stability of a gyroscope as a direction indicator with the datum

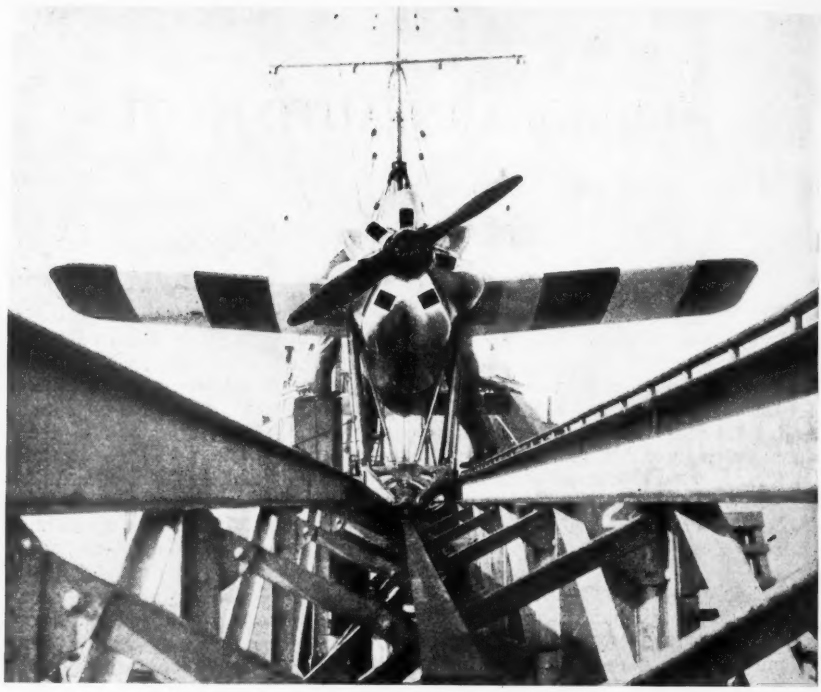


Fig. 1.
The Larynx on its launching catapult. (*Crown Copyright.*)

precision of a magnetic compass, a principle later applied in the D.R. compass, the Gyrosyn and the more recent G.III and G.IV compasses.

The "Larynx" controls were almost identical with those used some sixteen years later in the German V.1. It was appreciated at the time that the practicability of such a weapon was dependent upon the development of a cheap engine designed as an expendable store. It was also realised that London was the ideal target for such a weapon and that, as we as a nation were unlikely to be directing an attack against a target of suitable size at suitable range, having regard to the accuracy of wind prediction, it was more important to guard the secrecy of the project than to press the development of a suitable engine. The experience of the Second World War, in which Germany developed these weapons a few years too late to be decisive, vindicates the wisdom of the answer made by the Air Staff to the very nice question posed by the invention of this weapon.

Although the flying bomb project was pidgeon-holed, out of this early work the Mark 1A Auto-pilot and the Queen Bee target aeroplane emerged.

When the Mark 1A was born little was expected of auto-pilots beyond the capacity to stabilise the aeroplane on a moderately straight course and to execute gentle unbanked turns, for the purpose of correcting course and aiming bombs.

## GENERAL REQUIREMENTS OF A MODERN AUTO-PILOT

With the developments of modern aviation the requirements have become much more exacting. To-day a successful auto-pilot must be:—

(*a*) Capable of rapid correctly co-ordinated manœuvre.

(*b*) Able to maintain a compass course indefinitely.

(*c*) Capable of being coupled to radio devices to seek and follow a radio "track" and to follow a glide path beam for blind approach.

(*d*) Simple and fool-proof to operate.

(*e*) Safe in the event of any forseeable failure.

(*f*) Unneedful of frequent maintenance attention.

(*g*) Capable of rapid replacement by units to avoid dislocation of schedules in the event of failure.

Looking into the near future we find that it must also be capable of controlling aeroplanes with short periods of the order of one second, with little or no inherent damping and with unforseeable lags in the transmission system, including power-operated controls.

## REASONS FOR PREFERRING AN ALL-ELECTRIC SYSTEM

As these requirements became clear it was soon realised that an electrical signal system was essential to provide the required flexibility to meet the continually developing needs of radio guidance and the stability peculiarities of new designs of aeroplanes.

Moreover, experience with pneumatic auto-pilots had shown that condensation and freezing of moisture necessitated either a completely enclosed system, or bulky drying apparatus requiring considerable maintenance and that lubrication also presented serious difficulties, because of the sweeping action of the passing air.

Hydraulic systems had been used with some success, but in view of recent developments in the control of small electric motors it appeared that on the grounds of simplification of installation and maintenance, an electric power system was also to be preferred.

Accordingly our firm decided in the closing months of the late war to proceed with an automatic pilot employing an electric power as well as signalling system.*

To avoid, so far as possible, the contact troubles of electric systems, it was decided to use A.C. for the signal and power systems and, to line up with the trend of electric supply in aircraft, we decided to use 115 volt 3-phase 400-cycle supply. Only commutatorless motors were acceptable and it was decided to use induction type signal generators, except for a few potentiometers which are power driven and in which relatively heavy brush pressures could be employed.

In a system of such complexity it is probably inevitable that a number of relays should be employed. Fortunately these components had recently been produced in miniature form and hermetically sealed, thus providing us with a ready-made solution of this part of our problem without risk of contact failures.

The position on the motor side was not so favourable because no real effort had been made in this country to develop small low inertia induction motors of reasonable efficiency.

## THE DEVELOPMENT OF THE HYSTERESIS SERVO-MOTOR

We were attracted by the theoretical potentialities of the hysteresis motor and we were perhaps fortunate in that, when I had turned from fluid power systems to electric, I was lacking the conventional training of the electrical engineer. I was thus unaware that the hysteresis motor was little more than a laboratory freak with a limited field of application in the clock technique and that Steinmetz had proved conclusively that its efficiency could not be expected to exceed a few per cent. I may even confess that, when the idea of using a hysteresis motor occurred to me, I was unaware that such a motor had even been proposed before.

Ewing had shown that the magnetic hysteresis of materials could be compared by rotating circular cylinders of the magnetic material in the field of a magnet and measuring the torque reaction on the magnet. Since, during one revolution of the rotor, it goes through a complete cycle of magnetisation the work done can readily be deduced from the area of the hysteresis loop of the magnetisation curve. Since, also, the work done is clearly $2\pi$ times the torque, the latter is calculable from the magnetic properties of the rotor material and, most important, it is independent of the speed of rotation. In the case of a solid cylindrical rotor the computation is complicated by the non-uniformity of

* The design we produced is known as S.E.P.1 standing for Smith Electric Pilot Model 1. This design has recently been adopted for the Royal Air Force under the Service title Mark 9.

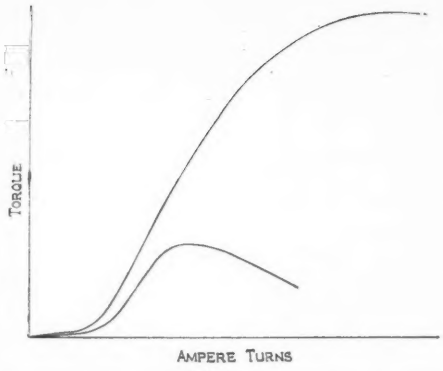



Fig. 2.

Relation between the excitation and the "pull-out" torque of an open slotted hysteresis motor for D.C. and A.C. excitation.

the intensity of magnetisation throughout the material. Moreover, the average intensity of magnetisation is necessarily low compared with the flux density crossing the air gap. For these reasons it is desirable to use a hollow annulus for the shape of the rotor.

It is obvious that rotation of the field magnet in the Ewing's machine will produce exactly the same torque on the rotor as rotation of the latter and if, instead of rotating physically the permanent magnet, we can use a polyphase stator to rotate the field electrically, the same result should be attainable.

On these grounds we should expect of a hysteresis motor a constant torque over the whole speed range up to synchronous speed and a reversal of torque on passing through the synchronous speed, thus giving a rotor loss proportional to the slip speed and therefore zero at the top of the speed range.

I had arrived at this stage in the analysis of the problem when my attention was drawn to Steinmetz' proof that the efficiency was prohibitively low.* Steinmetz lumped the efficiency and the power factor into a single criterion which he defined as "apparent efficiency." He demonstrated that the idealised "apparent efficiency" is $(1 - S) \sin \alpha$ where $S$ is the relative slip and $\alpha$ the angle between the axis of the resultant magnetism of the rotor and the axis of the rotating M.M.F. of the stator. From the materials available at the time he concluded that, in a magnetic circuit containing an air gap, $\alpha$ is small (a few degrees only) and that in consequence the "apparent efficiency" is low. My analysis led me to expect the efficiency loss at zero slip to be mainly the copper loss of the stator and since this is independent of frequency and therefore of synchronous speed, the efficiency should increase steadily with frequency. In the circumstances I refused to be discouraged by Steinmetz and by the experienced electrical engineers with whom I came into contact.

Experiments with a conventional open slotted stator soon convinced me that the case was not quite so simple as at first appeared. The synchronous torque was disappointingly small and, as the ampere turns were increased beyond a certain point, the torque decreased and became negative. This is shown by the lower curve of Fig. 2.

The upper curve was obtained by using D.C. excitation of the stator and measuring the pull out torque from rest. The disparity between these two curves immediately suggested the explanation, particularly as the D.C. excitation gave a torque greater than could be expected from the simple theory.

Considering an element of the rotor slipping in the D.C. excited stator, it is obvious that it undergoes a number of subsidiary hysteresis cycles in passing the open slots as well as the fundamental cycle. Clearly the torque must suffice for the work done in the subsidiary cycles when the rotor is rotated in the D.C. excited field. But when the rotor is running at synchronous speed in the A.C. excited stator, the power associated with the subsidiary loops is being absorbed as rotor loss. Thus designating by $Q_F$ the torque due to the fundamental magnetisation loop and

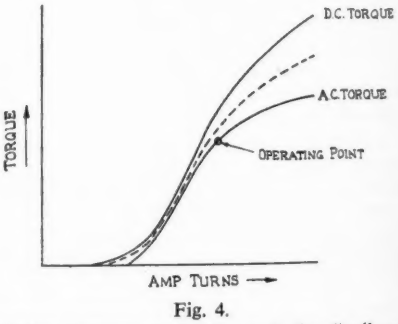



Fig. 4.

Relation between excitation and the "pull-out" torque for a hysteresis motor with "bridged" slots. Compare Fig. 2 for effect of open slots.

* "Theory and Calculation of Electrical Apparatus" by Steinmetz (1917).

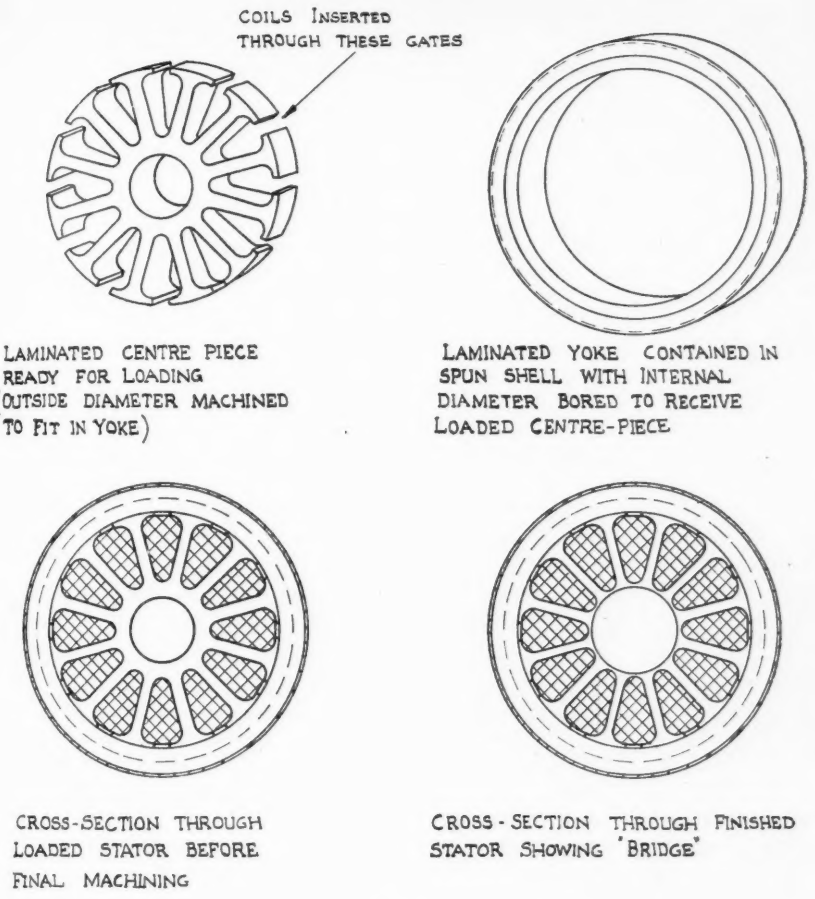

COILS INSERTED THROUGH THESE GATES
LAMINATED CENTRE PIECE READY FOR LOADING (OUTSIDE DIAMETER MACHINED TO FIT IN YOKE)
LAMINATED YOKE CONTAINED IN SPUN SHELL WITH INTERNAL DIAMETER BORED TO RECEIVE LOADED CENTRE-PIECE
CROSS-SECTION THROUGH LOADED STATOR BEFORE FINAL MACHINING
CROSS-SECTION THROUGH FINISHED STATOR SHOWING 'BRIDGE'

Fig. 3.
Stages in the production of the stator with bridged slots for a hysteresis motor.

by $Q_P$ the torque due to the auxiliary or parasitic loops, we should expect the two curves of Fig. 2 to be

$$Q_F + Q_P \text{ and } Q_F - Q_P \text{ respectively.}$$

To test this hypothesis, the effect of increasing the air gap was explored, since this would relatively reduce the intensity of the parasitic loops. It was found that this caused the D.C. and A.C. curves to approach as expected but, of course, at the expense of a prohibitive increase in the stator copper loss.

We then tried the effect of a thin iron sleeve bridging the open slots. This was also found to be effective.

Finally, to reduce the eddy current loss in the sleeve and also to provide an easier stator to load, we adopted the device of using externally slotted laminations allowing the coils to be loaded from the outside, the stator iron path being provided by a laminated annular ring enclosing the stator after loading. The bridges across the slots were then reduced to the minimum thickness, giving satisfactory reduction of the rotor parasitic loss by boring the stator to its final dimension after loading.

The stages in the production of the finished stator are shown in Fig. 3.

With this form of construction the disparity in the D.C. and A.C. torques was reduced, as shown in Fig. 4.

The dotted curve on Fig. 4 shows the value of $Q_F$ on the assumption that the parasitic loss is the difference between this curve and either of the others. It will be seen that the parasitic loss is only a few per cent. of the output power at the operating excitation, which is a very satisfactory result.

Figure 5 shows output power against input power for rotors of several degrees of magnetic hardness in the same stator, from which it appears that there is an optimum induction for each material and an optimum material for each output torque per unit of rotor volume.

From such data we have been able to design hysteresis motors for 400 cycle actuation, varying in output power from about one watt to about 80, with efficiencies of the order of 70 per cent. for powers in excess of 10 watts.

These motors generally have a higher torque and power absorption at standstill than at synchronous speed, due mainly to eddy currents in the rotor which are not unimportant at slow speeds. Fig. 6 shows the main performance data of the actual four-pole hysteresis motor used in the S.E.P.1 servo-motors running on a 400 cycle 3-phase supply. An efficiency of the order of 70 per cent. is attainable with a two-pole stator.

Before leaving the subject of the power units, I would like to point out an important advantage which the hysteresis motor has in this application.

A conventional induction motor of the squirrel cage type, when used as a servo-motor, is designed to give maximum torque at standstill for fairly obvious reasons. This results in a curve of torque/speed of the form marked $S_Q$ in Fig. 7. The hysteresis motor as shown in Fig. 6 gives a curve of the form marked $H$.

At standstill, obviously, both machines have zero efficiency. Nevertheless we are concerned with the economy with which the input power is converted into torque. It is therefore convenient to introduce a concept which, for lack of a better term, we may call "torque efficiency," by which I mean the ratio of the product of the torque and the synchronous speed to the input power. This figure, when multiplied by the ratio of actual speed to synchronous speed, which we may call the slip efficiency, is the actual efficiency. Now the "torque efficiency" of a hysteresis motor of the size used in our pilot is of the order of 60 per cent. and I am fairly confident in saying that it is impossible to design a squirrel-cage motor of comparable size with a better "torque efficiency" at standstill. Thus if, as in our case, the motor must be

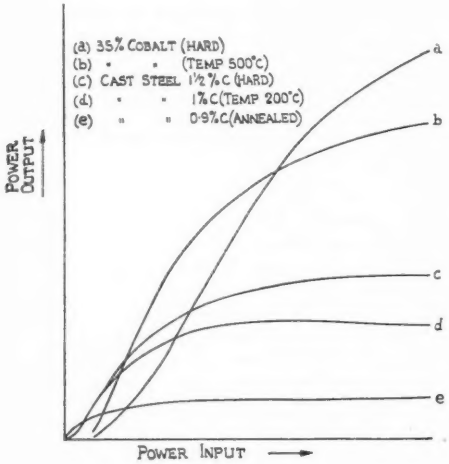



Fig. 5.

Effect of varying the magnetic hardness of the rotor material of a hysteresis motor.

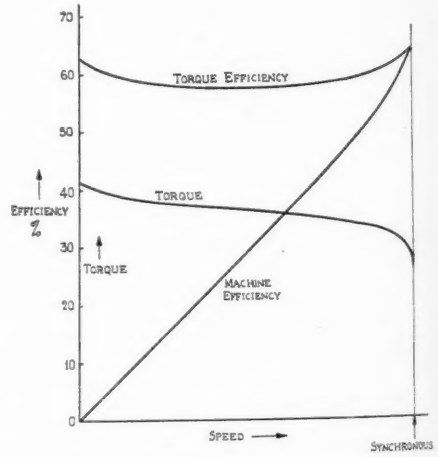



Fig. 6.

Performance data of a typical hysteresis motor designed for a small servo-motor.

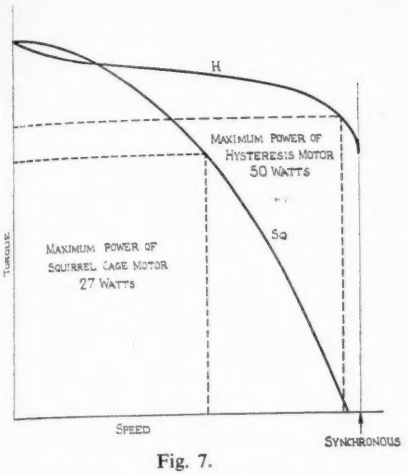



Fig. 7.

Comparison of torque/speed curves of a squirrel-cage and a hysteresis motor.

capable without overheating of sustaining full torque indefinitely, to allow for expected mishandling, both machines must be rated to give substantially the same standstill torque, as shown in the Fig. 7. It is obvious from the curves that the power available from the hysteresis motor, *i.e.* the maximum product of torque and speed, is considerably greater. Thus when designing to a specified power, the hysteresis motor permits an economy in the size and weight of both the motor and the amplifier supplying it.

We have not designed a hysteresis motor with a rotor of higher hysteretic power than 35 per cent. cobalt magnet steel, because of the difficulty of dissipating a greater power at standstill, but it may be noted that materials are available, with about five times the hysteretic power of 35 per cent. cobalt, which could be used in applications where rapid acceleration is a dominant requirement. Since one of our cobalt rotor machines, designed to give 12½ watts output, accelerates from rest to 24,000 r.p.m. in 1/10 sec., it is clear that a really spectacular acceleration is attainable by the use of such a material as Alcomax III.

## THE AMPLIFIER

An all-electric auto-pilot clearly requires an amplifier to convert the low level signals to motor power. So far this has necessitated either electronic amplification or relays, the latter being probably limited to a D.C. system. The German firm of Siemens produced, in their K.23 single channel rudder control for fighter aircraft, a very elegant system using a relay amplifier. They used a polarised relay controlling two power relays to reverse the armature supply to a D.C. motor and fed the armature volts back suppressively to the input of the amplifier, so as to obtain a motor speed proportional to demand. Since the relays are incapable of giving a graded continuous output the system is necessarily in a state of continuous hunting, but the frequency, about 20 cycles/sec., is high enough to be filtered out in the armature inertia. To offset the error involved in relating armature volts to speed, arising from the resistance drop in the armature, the armature current was fed back regeneratively to the input of the amplifier. By this device the motor speed was related to signal demand to a precision which was only limited by the variation of armature resistance with temperature. If required, temperature compensation could be employed to reduce this error. The K.23 also employed a saturable choke magnetic amplifier to mix the signals and drive the polarised relay.

It may be expected that the magnetic amplifier before long will be developed to the stage when it can be employed as the sole amplifying means in such equipment as auto-pilots. So far, however, it does not provide quite the flexibility of a valve amplifier.

In our A.C. pilot we have employed valves for the signal amplification and a magnetic amplifier for the power stage to drive the servo-motors, thus permitting all valves to be conservatively rated in the interests of reliability. Nevertheless the valves have proved the least reliable element in the whole system.

It is unfortunate that our manufacturers of valves cannot see their way to producing special valves for electronic equipment requiring a high order of reliability. There is a large field in industry for electronic control if the required standard of reliability could be guaranteed. The American valve manufacturers have seen this and are producing special valves for the purpose. Unless something is done about it soon, either the job will be done without valves or the art of electronic control will be in danger of becoming an American monopoly.

We experienced a number of cases of valve failure in which the valve ceased to function because of a crack in the glass seal.

To meet this contingency we adopted the policy of parallel circuiting so that no single simple valve failure could interfere with the overall functioning. Since making this change we have experienced several cases of internal shorting between the valve electrodes, due to faulty manufacture. This is a far more serious fault, because although we have been able to make our pilot "fail safe" in the event of open circuiting anywhere in the system, including the valves, it is not so easy to achieve the same degree of safety in the event of a short circuit from grid to anode, for example, which results in a reversal of sign of the A.C. transmission through the valve.

We have been forced to the expedient of testing all valves for interelectrode spacing by tapping them with 1,000 volts across each interelectrode gap and subsequently subjecting them to violent vibration while functioning normally. So far these tests are causing a rejection of about 20 per cent. of all valves received.*

It is particularly unfortunate that the required standard of valve manufacture is not available because a properly made valve, correctly circuited, is capable of giving reliable service for many thousands of hours and no other electrical device can touch it for sensitivity and speed of operation.

The magnetic amplifier inevitably involves an appreciable time lag and an appreciable input of power from a source of fairly low impedance. By accepting a very low gain factor for the input stage, and by such devices as positive voltage and suppressive current feed-back across stages, the lag can be greatly reduced. Moreover, in certain applications the lag, although gross compared with a valve amplifier, can be tolerated.

In our case, since we had decided to feed back servo-speed against the demand, as in the Siemens' K.23, instead of the conventional servo position feed-back, we were able to tolerate lag in the amplifier which would have led to servo hunting with the other system. Thus we were able to use a single stage of magnetic amplifier with simple regenerative current feed-back to step up the power level from one watt out of the valve amplifier, to over 100 watts into the hysteresis motor.

The conventional method of coupling an amplifier to an A.C. motor is to supply one phase direct to the motor and to supply a second phase in quadrature from the output of the amplifier. Such a system is uneconomical in that the power of the reference phase is being dissipated continuously in the motor, even when the torque demand is zero. It is true that such a system can be improved by a somewhat complicated arrangement to vary the reference phase also, according to the demand.

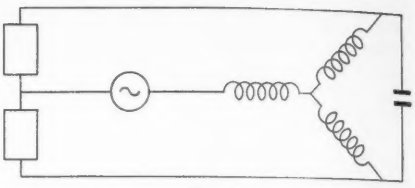


Fig. 8.

Method of circuiting a three-phase reversible motor from a single-phase supply.

We have preferred to use a three-phase winding with a single-phase supply and a phase splitting condenser across two phases of the motor. By connecting the single-phase supply between the third motor phase and one or the other side of the condenser, the torque of the motor can be readily reversed. Such an arrangement can give a perfectly balanced three-phase supply to the motor if the power factor of the motor windings is 50 per cent. The hysteresis motor used has a power factor of about 35 per cent. and the balance is good enough for all practical purposes.

The circuit is shown diagramatically in Fig. 8, in which the two rectangles represent impedances comprising saturable inductances, either of which can be reduced substantially to zero by differential control of two opposed D.C. currents supplied by the double triode discriminator valve which terminates the pre-amplifier. To save weight a single compound iron structure is used with a single balanced pair of control coils and the impedance coils enclose separate parts of the iron circuit.

A pair of parallel connected coils is used for each of the variable impedances and selenium metal rectifiers are used to generate a local circulating direct current in each such pair directly proportional to the A.C. current passing through the impedance. Thus positive feed-back is provided by the unbalancing of these circulating D.C. currents augmenting the control current. In the balanced state only a small current is

* Our valve rejection rate from all causes is about 50 per cent.

drawn through the two impedances, but the resultant small circulating D.C. currents polarise the core so as to discriminate the sign of any subsequent unbalance in the control windings.

So far as I know, this method of coupling a magnetic amplifier to a motor has never been used before. It has proved effective, relatively light and highly efficient.

## SAFETY REQUIREMENTS

I am sure it will be generally agreed that the most important requirement of an automatic pilot to-day is assurance that it will not endanger the structure of the aeroplane by dangerous control movements. The possibility of dangerous control movements must be considered under two separate headings:

(*a*) False control movements resulting from malfunctioning;

(*b*) Dangerous movements resulting during correct functioning from disturbances forced on the aeroplane by rough air or manœuvre.

Various suggestions, generally involving accelerometers, have been made for auxiliary safety devices to deal with both contingencies. These suggestions have been under investigation in Great Britain for about 20 years and more recently they have been considered again in relation to power-operated controls. One is forced to the conclusion that only strain gauges carefully placed on the aeroplane structural members expected to initiate failure can really provide a solution on these lines.

However, without such strain gauges, the situation is not as serious as might at first appear, provided that the danger of gross control movements caused by malfunctioning can be ruled out.

All aeroplanes must be designed with reasonable factors for handling by the human pilot in gusty weather. An automatic pilot, by virtue of its quicker response, can control with smaller movements than the human pilot. Moreover, when one considers, for example, the load on the tail structure, due to gusts acting on the tail organs, it is evident that in this case rapid correcting control movements reduce the structure loads.

Aileron loads are potentially dangerous, because of the rapid rolling accelerations which can arise in gusts and because the aileron may fail independently of the main wing structure.

This case also illustrates the inadequacy of accelerometer safeguards, since the rolling acceleration may well be zero or even in the opposite direction to that caused by the dangerous aileron loads.

In my opinion generous factors must be allowed on aileron strength until strain gauges are introduced to give warning when the stresses are becoming dangerous.

The Air Registration Board have covered the strength requirements in British Civil Airworthiness Requirements, Section D, Chapter 6 - 4, by the following paragraph:

"5.2. The inherent characteristics of the automatic pilot, as installed, shall be so related to the strength of the aeroplane that it will not, at any speed at which it may be used, cause controls to be applied in such a manner as to overstress any part of the aeroplane."

This is a very broad statement of a requirement. It is hoped that further investigation of the possible stresses imposed on a controlled aeroplane may at some time lead to more specific requirements but, in the meantime, the A.R.B. have issued a letter explaining how it is intended to interpret this clause.

I cannot do better than quote *in extenso* from that letter because of its close reasoning and the evident desire it reveals to make an unclear situation as clear as possible.

"Par. 5.2 contains a far more nebulous requirement which has been causing difficulty in interpretation. As I see it, for an auto-pilot in which failure to safety has been accepted as an intrinsic feature, it is only necessary to consider the correct auto-pilot response to such disturbances of the aeroplane as might arise. These disturbances can, I think, be split into two types:—

(*a*) A disturbance independent of the stabilising surfaces, *e.g.* an outer engine failure, centre of gravity shift, or gust striking the nose of the aircraft. In these cases, the auto-pilot will apply control to give loads in the same direction as would the stabilising surface when the aeroplane had moved through an angular displacement due to the disturbances.

(*b*) An air load due to a gust acting on one of the stabilising surfaces tending to cause a disturbance of the aeroplane. In this case, the auto-pilot applies control to give loads cancelling the applied air load.

"In case (*a*), it appears probable that an auto-pilot may, in checking the disturbance, apply greater loads than would occur in normal manual flight. These cases, however, are covered as regards stressing by the various manual flight manœuvring and gust cases which can, I feel, be accepted as providing a sufficient margin to cover the auto-pilot effect.

"In case (*b*), it would appear that an ideal auto-pilot would just provide control loads to cancel the disturbing load but that in practice the restoring loads may, due to any lag in the system, be greater than those necessary to balance the disturbing load. This increase, however, should be covered as regards total load by the manœuvring cases. It seems, therefore, safe to assume that the gust loads in auto-controlled flight will be somewhere between the free flight gust loads and the steady flight plus manœuvring loads. It only remains to check that the distribution of load, in the auto-controlled case, between the fixed and free control surfaces is also satisfactory. The auto-pilot control loads, applied by means of the movable surfaces, act in opposition to the disturbing loads on the fixed surfaces so the auto-control case will not be critical for the fixed surfaces, but it appears that it may be critical for the movable surface.

"As a quick, albeit approximate, method of proving that the movable surface will not be overstressed, the Board will accept evidence that the movable control surfaces have sufficient strength to withstand the air loads due to the maximum deflection at which the auto-pilot servo-motor torque can hold them at all speeds up to whichever is the lower of the auto-pilot placard speed or $V_{ne}$."

It thus appears that the critical conditions in stressing the aeroplane for use with an automatic pilot, functioning correctly, are the loads on the movable surfaces and that these must be of sufficient strength to withstand virtually the worst that the auto-pilot can do. In a subsequent paragraph a distinction is made between maximum load that the auto-pilot can apply and the maximum load it can sustain when back driven, but the concession on this account is so justifiably hedged around that for practical purposes we must take the maximum load which the auto-pilot can sustain.

However difficult the situation may be in relation to legitimate auto-pilot induced loads, it is manifestly desirable to limit, as far as possible, the risk of dangerous loads induced by malfunctioning of the auto-pilot. This means more than the aim of making the system as reliable as possible: it means ensuring that no foreseeable failure will lead to dangerous control movements. The A.R.B. have recognised the possibility of meeting this condition and have made it obligatory in future auto-pilot installations.

These considerations were mainly responsible for our selecting the rate/rate system whereby servo speeds are related to rates of deviation, instead of control movements being related to the deviations. This involved the use of three rate-of-turn gyroscopes, instead of the two displacement gyroscopes of the conventional art.

A displacement gyroscope can tumble if the rotor loses speed due to a bearing failure or an electrical fault. It can also tumble due to obstructions of the gimbal system or a gimbal bearing fault or, in extreme cases, because the rotor axis has become aligned with the outer gimbal axis, thereby reducing the degrees of freedom from three to two. Obviously tumbling of the gyroscope can lead to dangerous control movements in the case of a displacement control.

Our rate gyroscope is elastically suspended on leaf springs which are designed with an adequate factor of safety, thus providing complete immunity from tumbling complications. In the event of failure of the rotor speed, the gyro signal and therefore the control gearing simply decay to zero.

Again, with a displacement control, temporary loss of function of the amplifier or servo-motor leads to storage of the demand and, when function is regained, this stored demand may lead to sudden dangerous movement of the control. With the rate/rate system the datum is lost in these conditions so that when function is regained the control picks up safely from a new datum.

There is an insistant demand from some quarters that we should supply an automatic elevator trim motor. We have gone some way towards meeting this demand by providing a trim motor as an optional extra component, but we advise against its incorporation because it reintroduces the dangerous possibility of storage which has been eliminated in the auto-pilot proper.

Conditions might arise, because of mechanical obstruction of the control system or failure in the servo-motor, when the trim

motor may be required, on false information, to alter the trim up to the limit to which it is permitted to function. If the trim control takes the common form of a trim tab, the resultant pitch control is reversed so that the condition would be divergent. When the pilot became alarmed and resumed manual control, the situation would be unnecessarily complicated by the false elevator trim. We do, of course, supply a trim gauge whereby the pilot can detect any change of elevator trim when flying under auto-control.

The only plausible case that has been put up for an automatic elevator trim motor rests on the large changes of trim which are often encountered on lowering the flaps and undercarriage before landing. It is argued that, in a blind approach, the pilot, who may be overburdened with responsibility, may forget to make or order the required adjustment of trim. I would point out, however, that in at least two modern aircraft this point has been met by the aeroplane designer so that no adjustment of trim is required in these circumstances. This appears to me to be preferable to the proposal to adjust the trim automatically. In the meantime, the responsibility on the pilot for correcting trim manually, is not onerous and I submit that it is an unnecessary derogation of the pilot's authority to make trim changes without his sanction. Perhaps I am labouring this point too much, but I feel that a line must be drawn somewhere between justifiable demands for automatic control and gadgets for gadgets' sake.

## RATE/RATE CONTROL

There is often considerable confusion regarding the difference between a rate/rate control and a conventional displacement control. Ideally they are identical, because, if the simple equation,

$$a\zeta = \psi \qquad \text{[Displacement]}$$

is satisfied, it follows that the first derivative equation

$$aD\zeta = D\psi \qquad \text{[Rate/Rate]}$$

is also satisfied.

The differences occur when we consider the effect of motor inertia and the methods adopted in each case for monitoring to control the datum against long-term deviation.

Consider first the question of motor inertia. The practical equations in the two cases, when misalignment controls motor torque through a high gain amplifier, are:—

$$ID^2\zeta = K\,[\psi - a\zeta] \qquad \text{[Displacement]}$$

and,

$$ID^2\zeta = K\,[D\psi - aD\zeta] \qquad \text{[Rate/Rate]}$$

It is clear that in the first case $a\zeta$ follows $\psi$ by an undamped oscillation characterised by,

$$\left[D^2 + a\frac{K}{I}\right]\zeta = 0 \qquad \text{[Displacement]}$$

and in the second case the free motion is represented by the simple subsidence,

$$\left[D + a\frac{K}{I}\right]D\zeta = 0 \qquad \text{[Rate/Rate]}$$

Clearly the displacement control requires some damping either in the motor or by the addition of a velocity signal to the amplifier input, whereas the second case is inherently stable.

It is difficult to avoid some slight lag in the closed loop involving amplifier, motor and feed-back so that, if the frequency of the displacement system is forced high to reduce the misalignment (*i.e.* $K$ large), the need for damping becomes more insistent. Now, whether the damping is introduced by direct power losses in the motor or by a speed feed-back signal, it inevitably introduces undesirable lag in the motor response unless it is backed off by a corresponding rate demand signal, in which case we shall have introduced all the elements of the rate/rate system as well as those of the displacement system.

It should be noted also that, if the servo loop is at all critical, adjustment of the gearing constant "a" can cause a servo hunt.

These difficulties are completely avoided by the rate/rate system, in which the gain constant $K$ may be pressed as high as we please to reduce the misalignment. Also, as explained earlier in relation to the amplifier, the system will tolerate an amplifier lag which would be prohibitive in a displacement system.

Regarding monitoring, I have often been asked if the rate/rate system is not inherently unsound since the unavoidable datum error of the system will be continuously integrated.

This criticism appears, at first, valid when we compare the rate/rate system with a displacement system based on a compass of the Pioneer Flux-Gate type, but it is manifestly invalid when we compare with the case of a directional gyro which also integrates inevitable processional rate errors.

In both cases there is a need for long-term monitoring. Thus, in the case of the rudder

control, the rate/rate system is monitored by an azimuth signal derived from a magnetic compass and the displacement system requires monitoring of the azimuth gyro again by a compass signal. Both systems introduce only slow rates in response to compass signal, so that short period errors of the compass are unimportant.

When the limitations of a displacement gyroscope are thus considered, it is apparent that even the Flux-Gate system is subject to the same limitations, because the Flux-Gate element requires stabilisation to the horizontal plane and this can only be achieved by a gravity monitored gyroscope. In fact, it may be argued that, since the vertical component of the earth's magnetic field is commonly some two or three times greater than the horizontal component, the degree of gyroscopic precision required is increased considerably.

It is true that a high degree of datum precision is more easily obtainable with a displacement gyroscope than with a rate gyroscope. The rate gyroscope of our pilot measures up to 10° per second or 36,000° per hour. Thus to achieve a datum precision of 36° per hour, a range of measurement of 1,000/1 is required, whereas a displacement gyroscope can be made, without difficulty, with a precession rate not greater than 15° per hour.

On the other hand, it must be appreciated that the high precision of the displacement gyro is dependent upon near perfection of the gimbal bearings and is easily lost in the rough usage to which aircraft instruments are sometimes inevitably subjected. The rate gyroscope, although inherently less precise, is considerably more robust.

The crux of the matter is that even if the precession rate of the displacement gyroscope could be reduced to zero, monitoring would still be required because of the earth's rotation. Thus, in both systems, the gyroscope must be considered as a dynamic stabiliser and not as a datum defining instrument.

We have encountered some difficulty in preserving the aileron gyroscope datum in turning flight. The difficulty became manifest in the experiments in blind approach made at the Blind Landing Experimental Unit. During the approach under the control of the radio signals, turning instructions are operative almost continuously for about five minutes and at the end of this time, when approaching the touch-down, an accurate lateral datum is required. We have arranged to inhibit the cross-level gravity monitoring whenever turning is demanded to avoid falsification by the centripetal acceleration. Thus, with a datum precision of, say, 36° per hour, some 3° of banking error was to be expected at the end of the approach run.

To overcome this difficulty two remedies were introduced, either of which would perhaps have sufficed. The first is an automatic balancer which, during straight flight, continuously and slowly accumulates a correcting signal dependent upon the sign of the gravity monitor signal. Thus if there is a gyro datum error demanding biased action of the monitor, it is being continuously corrected, so that the approach run may be started with the gyroscope balanced up to the last minute, instead of depending on the balance set up by the manufacturer. The second remedy was to discriminate between turns of less than, and more than, 5° bank angle. For the slower turns the gravity monitor is left in action as for straight flight and, for the steeply banked turns, it co-ordinates the turn by modifying the rate of turn. By ensuring that the unmodified co-ordination of rate of turn and angle of bank is set for the approach speed, the pendulum shows no centripetal acceleration error and can therefore be relied upon to monitor the aileron gyro. The manœuvre of closing the beam during which steep turns are demanded, only lasts about half a minute so that the aileron gyroscope is thus monitored practically throughout the approach run.

These improvements were incorporated experimentally into the B.L.E.U. models of

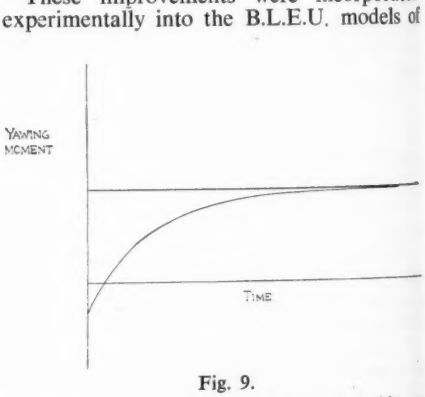



Fig. 9.
Yawing moment response of a Flettner rudder to a step function input.

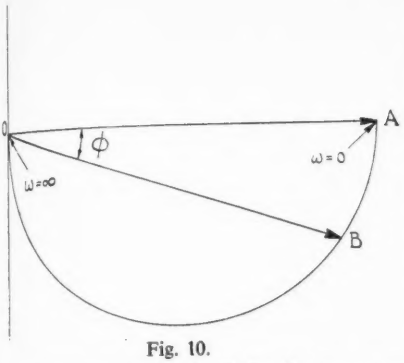



Fig. 10.
Transfer function for a simple lagging response.

our S.E.P.1 and will be incorporated as standard features in the version designated S.E.P.1.C., in which provision is also being made for barometric monitoring of height and radio control of tracking and approach.

## STABILITY AND EFFECT OF LAG

As aeroplanes increase in size and speed, it is becoming increasingly difficult to control them for several reasons.

In the first place the inherent damping is being steadily reduced, especially in yaw. The short period yawing motion approximates closely to a simple weather vane. Thus the reactive restoring moment is proportional to the dynamic stream pressure $\frac{1}{2}\rho V^2$. But the damping moment is also proportional to the angle of attack due to rotational velocity and is thus also inversely proportional to $V$. Therefore, the damping coefficient is proportional to $\frac{1}{2}\rho V$, and its significance, compared with the reactive moment, becomes steadily less as $V$ increases.

The resulting difficulty is increased by the substitution of jet propulsion for airscrews which contribute considerable damping, as a result of the difference in thrust on the two sides when the aeroplane is rotating in yaw.

To make matters worse, we are now faced with the problem of indirect operation of the rudder, with the certainty that this will introduce appreciable phase lag in the positioning of the rudder. This can be readily appreciated in the case of the simple Flettner rudder when we consider the yawing moment response to a step-function displacement of the control. The initial yawing moment response, when only the tab has moved, is negative and the desired positive response appears later, when the tab has moved the rudder against its natural damping to the required position. I am ignoring the additional lag due to the rudder inertia, because it is dependent upon the square of the ratio of the demand frequency to the natural frequency of the rudder and there is generally little difficulty in keeping this lag within tolerable limits. As a practical rule inertial lag may be neglected if the natural frequency of the mass concerned is not less than five times the demand frequency, which in the present case is the short period in yaw of the controlled aeroplane.

For practical purposes, therefore, the yawing moment response of a Flettner rudder to a step function input is of the form shown in Fig. 9.

The subsidence time of this response is easily calculated if the hinge moments due to rudder angle and rudder rate are known, but the effective lag is somewhat increased by the initial negative response.

The conventional spring tab control clearly contains this lag and it should be noted that the friction of the rudder hinge enhances the rudder movement damping as the amplitude is reduced, so that it is difficult to avoid sufficient lag to destroy all damping of the aeroplane motion at some small amplitude. Here the "set-up" spring tab appears to possess an important advantage over a simple Flettner control since, for a small amplitude at which hunting might be expected, the tab may be made inoperative.

Power-operated controls require close scrutiny in this respect. Non-linear lags, such as the effect of friction described above, are particularly objectionable because they are so unpredictable. The typical characteristic of hydraulic valve mechanisms can easily lead to trouble, since there may well be a "dead spot" before the movement of the valve reverses the movement of the motor. When the amplitude of an oscillation decays till it approaches the magnitude of the dead-spot, the phase of the motor response will lag by approximately 90°. This effect is reduced by valve leakage at dead centre. By judicious design, sufficient motor speed for the small amplitude we are considering, may be obtained by the unbalanced leakage while the valve is still working in the "dead spot."

In view of the potential difficulties arising, not only in the case of automatic control but also in the case of manual control, it is clear

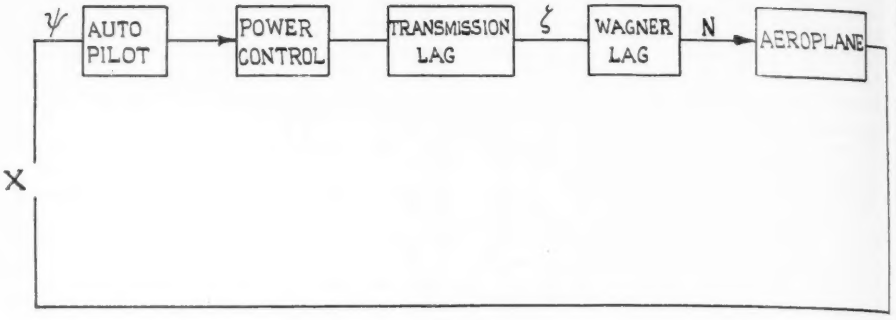



Fig. 11.
Block diagram of the yawing motion of a controlled aeroplane.

that no power-operated control should be accepted for flight test without a preliminary survey of its transmission characteristics over the range of frequencies and amplitudes with which we are concerned.

This transmission characteristic is defined by what is known as the "transfer function" for a simple harmonic motion demand.

In Fig. 10, suppose the demand to be represented by the vector $OA$ and the resulting output to be represented by the vector $OB$. Then the phase lag is represented by the contained angle $\Phi$. The transfer function is defined as the vectorial ratio $OB:OA$ which must not be confused with the scalar ratio representing only the amplification factor.

Now if the demand amplitude is maintained constant and the frequency is increased, the point $B$ will describe a locus somewhat resembling the semi-circle shown. If this curve is drawn to such a scale that $OA$ is unity and suitably marked with a frequency scale, it fully describes the transfer function of the system. The loci of the transfer function should be explored for a range of amplitudes from the greatest to the smallest with which we are concerned.

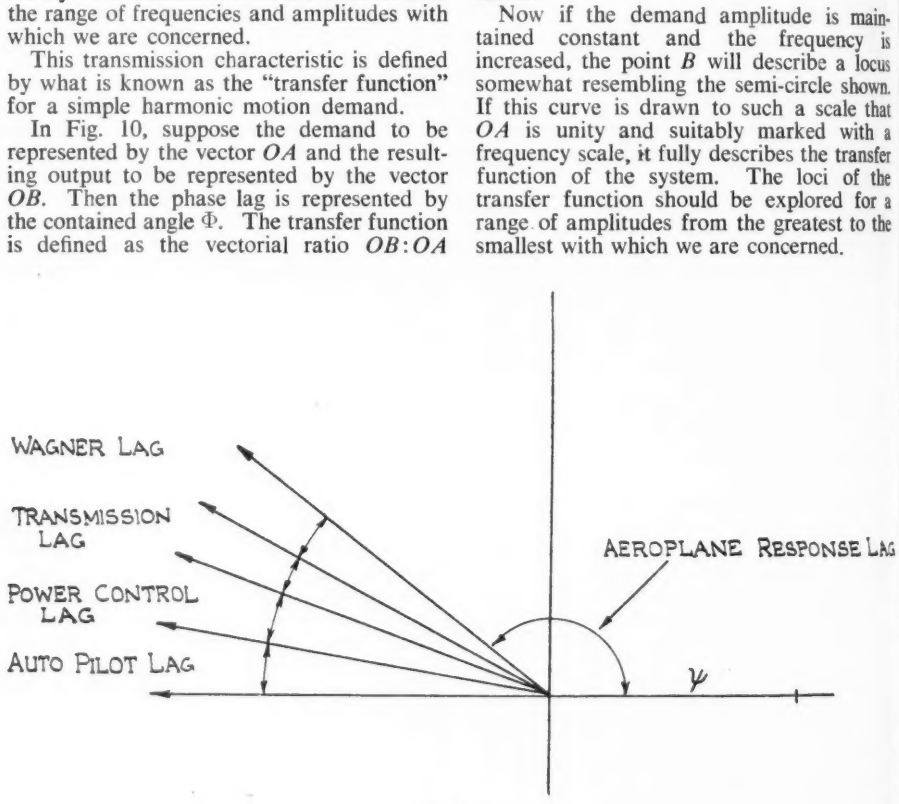



Fig. 12.
Condition for a stable sustained oscillation.

We are now in a position to consider the problem of the accumulation of lags round the whole system. Fig. 11 represents a block diagram of the yawing motion of a controlled aeroplane.

The stability of such a system is readily examined by the criterion due to Nyquist, which may be very simply explained.

Imagine the circuit to be cut at the point $X$ and a simple harmonic input signal $\psi$ to be supplied externally.

Then through each block the signal will be multiplied by the transfer function of that block so that, at the cut point $X$ emerges $F\psi$ where $F$ is the transfer function of the whole system and is defined by $F = f_1 f_2 f_3 \ldots f_n$ where $f_1$, $f_2$, etc., are the transfer functions of the individual blocks.

Now quite obviously if $F = 1$, the output $F\psi$ may be substituted for the input $\psi$ and the system will continue to oscillate, without external driving if the loop is closed. This condition is vectorially represented in Fig. 12.

Now starting with a system which will just sustain a steady oscillation, it is fairly obvious that, if the total lag is decreased, the motion will decay and if it is increased, the motion will diverge until different relationships prevail.

If we now ascertain the individual transfer functions for each stage, including the aeroplane itself, we may draw the transfer function for the complete closed system. In general it will resemble the curve of Fig. 13.

As sketched in Fig. 13, the system is stable, because it required an increase in the total lag to satisfy the condition of a stable sustained oscillation.

In this particular case instability could be caused by an increase of the overall amplification factor, *e.g.* by increasing the auto-pilot gearing or by increasing the speed of flight, but a warning should be given that in many practical systems the Nyquist diagram may take such a form that the system is stable over a range of amplifications, but unstable if the amplification is either increased or decreased beyond the limits of this range.

Mathematically it can be demonstrated that, if the complete transfer function locus be drawn for variation of frequency from $-\infty$ to $+\infty$, the system will be stable if the point $+1$ is encircled counter-clockwise and unstable if it is encircled clockwise. I believe that, stated in this form, the criterion is rigid and covers simple exponential divergent instability, as well as oscillatory instability.

Without getting involved in complicated mathematics, simple consideration of the effect of increasing or decreasing the total lag will indicate the proximity to the critical condition and whether or not we are on the stable side of it.

Before assessing the total tolerable lag in the case of a controlled aeroplane, we require

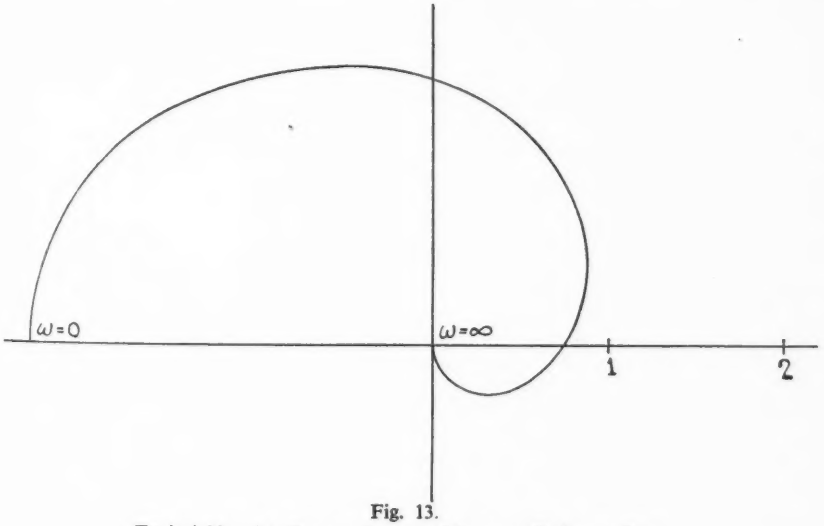



Fig. 13.
Typical Nyquist diagram for a stable controlled aeroplane.

to know the lag of the aeroplane. With the simplifying assumption, indicated earlier, that the short period yawing oscillation may be considered as the motion of a weather vane with, of course, means representing the rudder for forcing it, the relationships are shown in Fig. 14.

In Fig. 14 the control moment is derived as a leading vector from the displacement vector. The locus of the control vector as $\omega$ is varied is clearly a parabola.

Figure 15 shows, in the lower full line loop, the transfer function of the aeroplane in response to the rudder and how it may be simply derived from the parabola of Fig. 14. To invert the ratio, control moment: displacement, we must derive the reciprocal vector—the upper dotted loop and reflect this curve to transfer phase lead to phase lag.

I have reproduced this construction to facilitate judgment, by inspection, of the effect of reducing the inherent damping of the aeroplane. The relevant portion of the transfer function locus is the left-hand portion, between resonance and $\omega = \infty$, because the critical controlled frequency is always higher than the uncontrolled, unless the control law is extremely complicated.

Figure 16 shows four transfer function loci for degrees of damping varying from zero to well damped. The symbol $\mu$ is used to designate the relative damping referred to critical damping, $\mu = 1$. The symbol $r$ represents the ratio of operating to resonant frequency and constant $r$ contours are shown dotted.

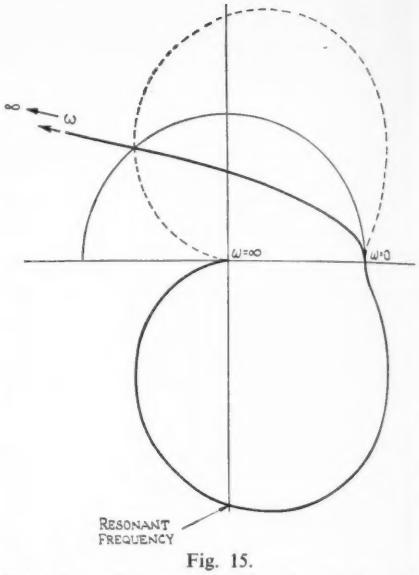



Fig. 15.

Derivation of the transfer function of the aeroplane considered as a simple damped oscillatory system.

It will be observed that the transfer function of the aeroplane always introduces a lag in excess of 90°, that the lag increases

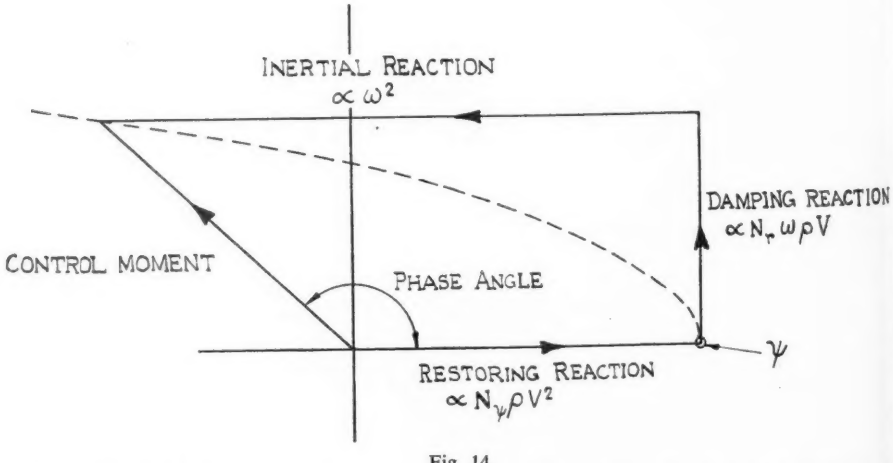



Fig. 14.

Vectorial relationship between the control moment and the resulting disturbance of a simple damped oscillatory system with simple harmonic forcing.

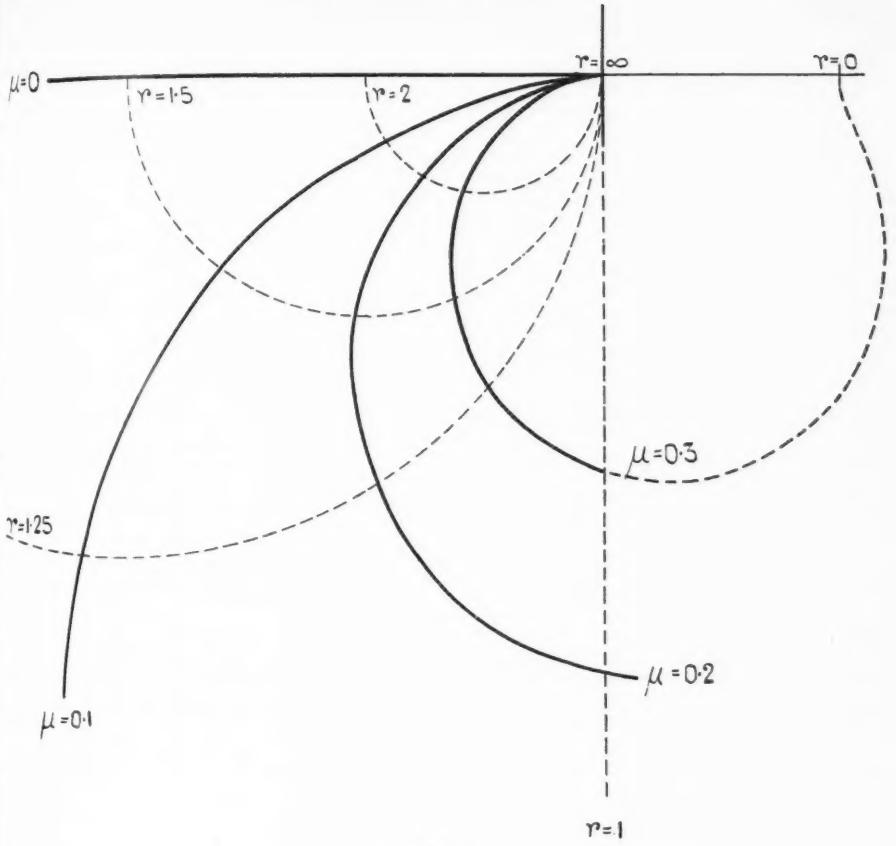



Fig. 16.
Aeroplane transfer function loci for various relative dampings.

but little from $\mu=0.3$ to $\mu=0.2$, but that it increases rapidly towards 180° as the damping approaches zero.

Now reverting to Fig. 12, it is immediately obvious that the sum of all the lags must not be allowed to reach 180°. Thus the advent of aeroplanes approximating to $\mu=0$ will require the strictest control of all the other lags. When, in addition, we are faced with a shortening of the yaw resonance period, the magnitude of the problem becomes apparent. If, for example, we specify that the accumulated phase lag, apart from the aeroplane, is not to exceed 12°, this will be 1/30th of the controlled period, *i.e.* 100 milliseconds for a three second period and 33 milliseconds for a one second period.

It is frequently suggested that this difficulty should be met by designing the auto-pilot to give a phase lead instead of a phase lag. There are several reasons why it is undesirable to rely too much on this remedy.

In the first place the result may be to produce aeroplanes which, while quite satisfactory under automatic control, will only be controllable by the human pilot when he has acquired sufficient skill to introduce a similar phase lead. This anticipation becomes increasingly difficult as we shorten the period and it may well be that with periods approaching one second, we shall be straining the human factor too far.

It has been suggested that, provided the uncontrolled aeroplane is well damped, this

difficulty will not arise, because the human pilot will not attempt to control the short period. I think this opinion is invalid because close control demands precise response and, even if it be true that the human pilot can avoid a divergent oscillation by the simple expedient of doing nothing, he is unlikely thereby to achieve the precise result he desires. Moreover, the major premise of the argument that the uncontrolled aeroplane will be well damped is, in the light of recent experience, questionable.

Further, if the auto-pilot is designed to give a component of rudder displacement proportional to rate of yaw, the dynamic gearing will increase without limit with increase of frequency, so that the control becomes harsh to the transient disturbances of rough weather. If pressed too far, this undermines the confidence of the crew and unduly complicates the problem of stressing the aeroplane for rough weather flying.

Finally, if reliance is placed on the auto-pilot to correct the accumulated lags, it is essential that those lags should be reasonably constant with variation of amplitude. No practical phase advance could prevent small amplitude hunting if other parts of the system, through back-lash, friction or dead spots in hydraulic valves, introduce preposterous lags at small amplitude.

It is possible, and perhaps desirable, that the auto-pilot should introduce a large measure of phase advance for small amplitudes and let the first derivative term fade out for large amplitudes. Provided that care is taken to ensure that the system does not go unstable for large amplitudes, this would ensure rapid decay of small disturbances and increase the tolerance to threshold lags in the rest of the system.

Again, more complex transfer functions than simple first derivative addition may be provided by the auto-pilot.

Figure 17 shows three phase-advancing transfer function loci. Line *A* is for simple addition of the first derivative. Curve *B* represents the performance of a practical circuit in which, deliberately, the inevitable lag in the derivation of the first derivative is left fairly large. Curve *C* represents a function which can easily be obtained by various filter networks and which results in a reduction instead of a gain in the amplification factor between the static case, zero frequency and the dynamic case, controlled frequency.

We have recently tested in flight a phase advancing network giving the characteristic of curve *C*. It was tried out with our S.E.P.1 in the rudder channel of an aeroplane with a spring tab rudder which had manifested almost zero damping with a straight control. The results were extremely satisfactory. Not only did it produce what appeared to be aperiodic damping of any yawing disturbance but the pilot, who had great experience of auto-pilots, was of the opinion that the control movements in rough weather were noticeably reduced, in spite of the greatly enhanced control.

It will be observed that this phase advancing network results in a quite noticeable phase lag at low frequency. The time constants of the network are adjusted so that the frequency at which the phase begins to lead is well below the resonance frequency of the aeroplane. As I have shown in Fig. 15 the transfer function of the aeroplane has a phase lag of 90° at resonance, so that below this frequency the phase lags in the auto-pilot and the rest of the system may safely be allowed to approach 90°.

As a result of the tests we have made with this phase advancing network, we have decided to incorporate it as standard in the rudder channel of the new S.E.P.1.C.

It is to be hoped that the aeroplane designers will not, on this account, consider that the whole question of phase lags in controls may be shelved. Every effort should be made to eliminate the "threshold" lags and to keep the main lags down to a standard which I should like to see set at 0.025 of the shortest resonance period of the aeroplane. I have as yet no knowledge of the magnitude of the Wagner lag—the delay in building up the circulation round the fin and rudder after the rudder has moved. This would appear to be an irreducible minimum for the lag of the aeroplane control system.

## RADIO GUIDANCE

The problem of guiding an aeroplane along a radio beam presents a similar problem in stability, but concerned with periods in the range of 10 to 40 seconds instead of the range one to three seconds in the case I have discussed above. The longer period motion is amenable to the same methods and the broad result is that the beam displacement signal must be backed-off by another signal representing the rate of

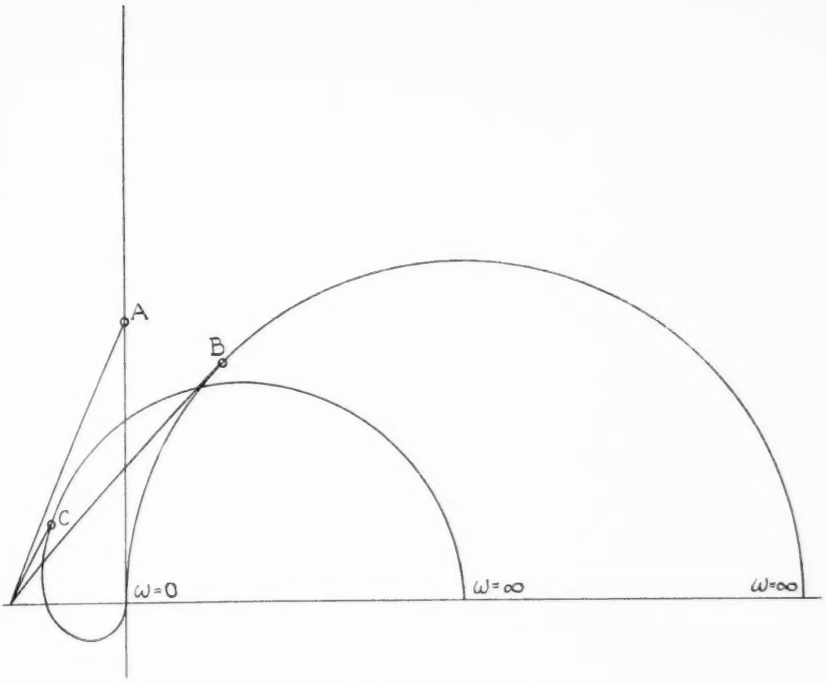



Fig. 17.
Three transfer function loci: A. Derivative addition; B. Lagging derivative addition; C. Second order phase advance network.

approach to the beam. This rate of approach signal may be obtained either from a compass repeater which has been set to the bearing of the beam, or, without any setting, be derived from the first differential of the beam signal electronically.

Both these systems have been tried out successfully by the Blind Landing Experimental Unit and I do not wish to attempt to anticipate the conclusions which they may reach.

We have been simulating the problem of closing and holding a beam in the laboratory and find conclusive evidence for the need for making the whole process automatic. It is difficult enough to hold the beam with sufficient precision but the problem of closing in a reasonably short time, using the information provided by compass and beam indicator, is far too difficult to impose on the pilot in the trying circumstances of a blind approach. It is really illuminating to attempt to execute the manœuvre, using the pilot's controller of the auto-pilot and then, when feeling frustrated, to switch in the auto-pilot radio switch and see one's mistakes being immediately corrected and the beam signal subsiding cleanly to zero.

I should like to express a purely personal opinion that perfectly safe blind approaches will before long be a matter of simple routine, using I.L.S. in conjunction with the auto-pilot simultaneously with a G.C.A. system supplying a running commentary to give extra confidence to the pilot and to provide a standby, in case anything should go wrong with the necessarily complex sequence involved in the auto-approach system.

## PROSPECT

The trend of modern life is towards increasing complexity and, as a result, the increased use of automatic devices to avoid overstrain of the human factor. To this rule the aeroplane is no exception. We have

reached, if not passed, the stage when the instrument panel is becoming a nightmare. Gradually automatic aids are coming to the assistance of the harassed pilot and we must look forward to the time when the automatic pilot will be the centre of a complete system of automatic flight control.

There are, however, two dangers which must be carefully watched—the danger that failures of the automatics may be more dangerous than the human failures they are designed to prevent and the danger that, through lack of practice, the pilots may lose the skill required in an emergency.

## DISCUSSION

**E. T. Jones** (R.D. Inst., Ministry of Supply, *Fellow*): Could the lecturer offer some reduction in weight now that he had developed all the components after such intensive development and research work? He disagreed with Mr. Meredith about the need for automatic trimming. When there were heavy stick loads which were suddenly discontinued, large accelerations were caused through the change of incidence. The more modern the aircraft, the higher the acceleration would become and he considered that automatic trimming was a good thing. It was also good for the rudder in cases of asymmetric flights with one or more engines dead.

Would the hysteresis motors fitted in unheated parts of the aeroplane, and subject to low temperatures, suffer considerable reduction in torque? Mr. Meredith had implied that he believed that eventually all landings, whether made in good weather or bad, in daylight or darkness, would be made automatically. Would any of the young pilots present subscribe to that policy?

The S.E.P. weighed pounds and cost pounds and had to be maintained, but there were other points in its favour, apart from performance. He had seen it quoted that there was a saving of fuel of between one and five per cent., according to the length of the journey, as between controlled flight by an automatic pilot and controlled flight by a human pilot. Over a long distance that was a considerable saving in weight and cost of fuel. The potential stability of the aeroplane was increased by fitting an automatic pilot. One or two aircraft designers were present and he thought they would agree that they had suffered sometimes from a lack of stability.

Gust loads, information on which might be obtained by a vane sticking out in front of the wing, could be passed immediately to the auto-pilot and the controls operated to relieve the structural loads which would otherwise obtain. Lastly, there was no alternative method, anyway, of landing blind.

**H. B. Sedgfield** (Sperry Gyroscope Co. Ltd.): Mr. Meredith appeared to have lost faith in hydraulic fluid systems and, although not fundamentally an electrical engineer, had transferred his affections to the hysteresis motor. As an electrical engineer, he felt that there were powerful arguments for the hydraulic servo for certain applications. To cope with modern auto-pilot requirements, it was imperative to obtain a useful horse power output from the servo and electric servos were, to a certain extent, limited in that direction since weight was of such importance. By using high pressure hydraulics, it was possible to obtain a power output three or four times that available from an electric servo for a weight of perhaps half, and even less, that of a corresponding electric servo.

Mr. Meredith also appeared to be somewhat apprehensive of electrical contacts. Before the war and the days of power cuts, there was evidence that many electrical systems, such as power supplies, electric trams, trolley buses and trains, automatic telephone exchanges, and so on, ran with the greatest reliability, in spite of the fact that they were employing thousands of contacts. It seemed to him that an electrical contact, if properly designed, was perfectly reliable. Contact engineering was to-day a science of its own, a fact appreciated by the Germans and others, during the last war. Thus to go over to A.C. purely from the point of view of avoiding electrical contacts appeared to throw away some of the advantages which D.C. systems might have in cases where weight was at a premium.

The lecturer appeared to indicate that the rate/rate system was the best way to stabilise the servo system. That seemed to him to be a little questionable as it was only

one way of reducing servo lags. With the so-called displacement/displacement system, servo stability could be attained by the incorporation of an error derivative without introducing servo lags in the case of a virtually undamped servo load; and with viscous load damping present, any servo lag could be counteracted with an input signal derivative term. While in America recently, he had asked the designer of the A.12 automatic pilot (a displacement/displacement type) if he intended to improve further the response of the servo system, and had the reply that as the performance was already adequate it seemed unnecessary. With a throttle-controlled hydraulic system, it was possible to obtain a perfectly stable response, a high speed response, which was not subject to appreciable lags because as the valve closed it put up a very high damping effect on the servo which stabilised it at the very moment it was required to be stationary; when the valve opened there was little lag and the system equation approximated to a simple, and short, time constant. That was another argument in favour of the hydraulic servo.

He was rather surprised to learn that the S.E.P.I.C. was only now adopting phase advance circuits. This method of stabilising an aircraft had already been used with considerable success and was desirable even if it were possible to meet the ideal of having a servo with no lag at all. Even with an ideal lagless servo, if the stiffness (or gearing) of the automatic pilot were increased beyond a certain point, the natural aerodynamic damping present in the aircraft would not be adequate to give a critically damped response. If a very stiff automatic pilot control to a datum were required, they were almost inevitably forced to the use of phase advance networks, which had to be good ones.

He agreed with the lecturer on the subject of valves manufactured in Great Britain. Valve manufacturers were certainly not devoting sufficient attention to the production of reliable valves. With components which, after all, did not cost so very much, this was regrettable since the electronic valve possessed considerable advantages in lightness and completely negligible time delays. A certain amount of research which, presumably, could only be afforded by the Government, would be justified in an effort to produce reliable valves. Valves had been known to operate for 50,000 hours continuously without switching on and off, and a major trouble was associated with crystalline growth in the filament due to switching.

**F. W. Irons** (Standard Telephone and Cables Ltd.): The valve industry in Great Britain was quite capable of designing and manufacturing valves similar to the special American ones mentioned if an adequate demand were made clear to them, but not, as usually seemed to be expected, at the same price as valves of the normal radio type used at present. Work of that nature was, indeed, in hand.

Mr. Meredith's references to valve failures was probably not unrelated to the fact that certain of the valves which were presumed to be faulty were being operated at some 25 per cent. above their peak anode voltage and 30 per cent. above their peak anode current, as given by the manufacturers.

He would not guarantee that treatment such as the inspection which the equipment manufacturers applied to the valves, mentioned by Mr. Meredith, would not cause permanent injury. The application of more than twice the voltage for which the valve was designed, with violent tapping, might cause permanent harm to such a delicate mechanism.

**E. J. Richards** (Vickers-Armstrongs Ltd., Weybridge, *Assoc. Fellow*): When high altitude and high speed aircraft were considered the damping terms were very poor. There was no doubt that in order to get really satisfactory directional stability characteristics they must put in some kind of artificial damping term in the automatic pilot. Whereas the old air liners had high damping because of big fins, new jet air liners flying at high speeds and high altitudes were likely to be poorly damped; he was, therefore, pleased to hear that Mr. Meredith, in his next auto-pilot, the S.E.P.I.C., was putting in that high damping term in terms of a phase advance. What were the limitations to this arrangement? Did it mean that if there were a certain amount of backlash, the phase advance terms would be cancelled out, so that instead of getting any damping, a serious lag would occur?

The automatic trimming tab was not only useful to guarantee trim when changing from auto-pilot to manual control; in some cases it made the difference between the necessity for a duplication system and managing with a manual reversion system. Which was worse —the possibility of the automatic trimming tab sticking in the wrong position, or the

need to have a complicated duplication system? He was not at all convinced that the duplication system was the better answer.

Since the maximum load when the pilot reverted to manual in an emergency would be greater than the auto-pilot could cope with, did that mean that the automatic pilot could not be made to operate satisfactorily, or did it not matter that the auto-pilot might slip for a very short time? He had heard that when the automatic boost failed on Constellations coming over from the United States they wallowed round in the sky because the auto-pilot could not be used and that made things worse.

**J. G. M. Pardoe** (Air Registration Board, *Assoc. Fellow*): There were two main ways in which an auto-pilot could lead to danger. Up to the war period, when an auto-pilot went wrong it could put the aeroplane into a most unpleasant attitude. The case might be remembered of a flying boat on trans-Atlantic delivery which reached the headlines of the newspapers because of an error in the automatic pilot. Quite a number of aircraft had had mishaps in that way. The other possible fault was that the auto-pilot could go wrong and cause a control movement which was large enough to break the control surface, or the surface to which it was attached, before the aeroplane achieved an unpleasant attitude. That did not occur in the past because the pilot did not have enough power. The power had since been increased. They must be careful to see that that did not happen.

Valve manufacturers had done a good job in producing a tempting tool but had not "designed in" the reliability needed for aircraft. He hoped it was not too much to ask that they should go further now and make valves suitable for serious purposes. They might almost be grateful to them, however, for the failures they had produced, because otherwise Mr. Meredith's cleverness and the excellence of his pilot might have led designers into considerably more detailed response and stress calculations to take advantage of an assumption of utter infallibility. Perhaps it had saved a case of running before walking.

He crossed swords with Mr. Meredith on the question of strain gauges to prevent failure of the aeroplane. He would rather see an auto-pilot which did not need a protective device and he would have thought that Mr. Meredith agreed with him in that. His feelings were rather the same on trimming except where automatic trimming was necessary. It introduced a lot more risk of failure. It was not only a question of the trim motor stopping but of a wrong signal being fed or a failure giving more trimming than was wanted, or was safe.

Automatic approach was being done in this country and in America and some airline aeroplanes had the equipment. But he thought it was still one of those things which could be demonstrated on a fine day or once or twice on a foggy day, but the airlines were still a long way from being able to trust lives to it. The human pilot must still be able to do the job; if he liked to play with automatic approach to relieve fatigue, that was all it was fit for at the moment, but he would have to keep a very sharp watch all the time, particularly unless there was an improvement in valves.

## MR. MEREDITH'S REPLY

**Mr. Jones:** He wished he could say they could get on with the job of reducing weight, but he felt that Mr. Jones' major premise was wrong; they had not yet solved all the difficult problems. Even now they could not say for certain exactly what they would have to put into S.E.P.1C. in order to deal with the new aeroplanes coming in to service during the lifetime of the next design. They would do all they could to reduce weight but at the moment function and safety must be given first consideration.

Low temperatures did not have any effect on the hysteresis motor. Mr. Jones raised the question of all landings in all weathers being done under automatic control and, he thought, had attributed to him a suggestion that that should happen. In the wording of his paper he had used the word "approaches" rather than "landings" and he wished to maintain that distinction.

**Mr. Sedgfield:** He did not think he had lost faith in hydraulic systems; and he agreed with most of what Mr. Sedgfield had said. There were applications in which it had obvious advantages over electric systems. For the power sizes he was dealing with in the auto-pilot the A.C. electric solution appeared attractive or he would not have chosen it, but if he were working in the field of power-operated controls involving large power he would not necessarily choose the same solution.

Perhaps he had ruled out D.C. systems

and contacts too readily, but the trouble was that experience of contacts and commutators during the war had led to a considerable measure of distrust and they had to consider not only what they could do in a short time when they started development at the end of the war, but the confidence they could expect people to have in the product when they had produced it, and he thought that they had rightly eliminated contacts from the vital function of the auto-pilot. With improving technique they would be able to bring contacts back and he would be the last to say that they should forget all about potentiometers for all time.

He had not intended to convey the impression that the rate/rate system was the only satisfactory way of stabilising a servo motor. The observation which he had made in his paper, and which he repeated, was that with a displacement system, by the time they had dealt adequately with the problem, they had introduced all the elements of the rate/rate system, so they had the complexities of both systems.

He had indicated in his paper that there were several reasons why it was undesirable to place too much reliance on phase advance. Basically the reason he had not employed phase advance earlier was that the complexity could be avoided with the majority of existing aeroplanes and that he did not require it to cover up the deficiencies of a lagging servo system. This policy had been justified by the results, but it was recognised that aeroplanes were now contemplated which would require additional complexity in the auto-pilot on account of their short periods and lack of damping.

**Mr. Richards:** The main limitations of artificial damping introduced by phase advance in the auto-pilot appeared to be, (*i*) The very large control movements which resulted from such an arrangement when the aeroplane suffered a transient disturbance, (*ii*) the effect of a large damping derivative in shortening the already very short pitching period in certain proposed new aircraft, (*iii*) the inability of the device to do anything to improve the situation if there were an appreciable spatial lag, *i.e.* backlash, in the transmission or the power-operated control.

Bearing these limitations in mind, if aeroplanes with uncontrolled pitching periods of the order of $1\frac{1}{2}$ seconds and a very poor threshold of response in the power-operated control of the elevator had to be flown by an automatic pilot the problem would be far too difficult to be dealt with by known forms of phase advance in the auto-pilot.

He had thought that he had made out a good case for not taking the risk of dangerous false trim setting resulting from an otherwise safe failure in the main servomotor. He was prepared to agree that automatic trim as part of a power-operated control system was in a different category, because failure in the power-operated control was a risk which generally could not be accepted and, therefore, such safeguards must be available as would ensure that the trim motor would not be supplied with a false demand for a trim change.

He considered it dangerous to generalise without detailed consideration of all the systems envisaged, but he would suggest that the problem in this case was inherently related to the safety requirements of the power-operated control and that any provision for automatic trim should be part of the power-operated control system and in no way involve the auto-pilot.

**Mr. Irons:** He was glad to hear that the valve industry of this country was quite capable of designing and making valves to the standard of the American Red Series. He could assure Mr. Irons that he would not expect them at the same price as the normal radio type. The position was that the cost of screening the present valves and the servicing cost of dealing with failures in the field would allow a very handsome margin on the price of a really reliable article.

Mr. Irons had qualified his statement by the proviso "If an adequate demand were made clear to them." This remark appeared to him to reveal a lack of appreciation of the situation. The designer of electronic equipment was tied down by the limitations of the reliability of existing valves and was therefore unable to tap the potential market for such equipment. If the British valve industry could not take the responsibility to meet the obvious need on their own initiative then, as he had stated in his lecture, either the art of electronic control would be in danger of becoming an American monopoly or many of those problems would be solved without the use of valves.

Mr. Irons had suggested that certain of the valve failures to which reference had been made were due to over-loading. He regretted that Mr. Milsom had not contributed to this part of the discussion with his detailed knowledge of the circuit conditions. He could, however, assure Mr. Irons that the

figures he had quoted did not apply to the S.E.P.1 pilot. On the contrary, the valves were very conservatively rated and he attributed to this cause and to the elaborate screening adopted, the fact that the failure rate in service was extremely low compared with figures available from experience on the other side of the Atlantic. The burthen of his complaint was the lack of security against very rare, but sometimes dangerous, faults which might well be avoided by greater care in design and manufacture and closer inspection.

He agreed with Mr. Irons that in any screening tests, designed to reveal the presence of offending valves, care must be exercised to see that the treatment did not damage the sound ones. This aspect was receiving careful consideration.

**Mr. Pardoe:** He thought that Mr. Pardoe's disagreement with his views on strain gauges used to prevent aeroplane structural failure was more apparent than real since, in the lecture, he had intended to suggest that strain gauges appeared to be the only solution of the policeman type. He quite agreed that an honest citizenry was preferable to a police force but—to drop the analogy—if very large aeroplanes forced the issue of saving structure weight, then perhaps the strain gauge would have to be admitted in company with the gust reliever. He thought that if such devices were adopted they should be regarded as part of the aeroplane and not as part of the auto-pilot.

In conclusion he though that it was not the function of an auto-pilot to make an aeroplane structurally safe to handle, although it was quite reasonable to demand that the auto-pilot should not break an aeroplane which was intrinsically safe in the hands of a human pilot.

# RADAR AS AN AID TO THE STUDY OF THE ATMOSPHERE

by

F. E. JONES, M.B.E., B.Sc., Ph.D., A.M.I.E.E.

The 765th Lecture was read before the Royal Aeronautical Society on Thursday 27th January 1949, at the Institution of Civil Engineers, Great George Street, London, S.W.1. Dr. H. Roxbee Cox, B.Sc., D.I.C., F.R.Ae.S., F.I.Ae.S., President of the Society, introduced the lecturer, Dr. F. E. Jones, M.B.E., B.Sc., Ph.D., A.M.I.E.E., who has specialised in experimental physics and is Senior Principal Scientific Officer, Telecommunication Research Establishment. During the evening Dr. Jones demonstrated some of the apparatus discussed in his paper.

SUMMARY

Apart from its normal functions in aviation the applications of radar have been few. In the post-war period T.R.E.,* in conjunction with the Meteorological Office, have been making a study of possible uses of radar to the science of meteorology and the results of these investigations are described in this paper.

In the experimental work radar has been used to:—

(*a*) investigate precipitation,
(*b*) detect clouds dangerous to flying,
(*c*) measure cloud height, and
(*d*) measure wind speed and direction.

An account is given of how radar may be used to measure the density of the atmosphere at altitudes up to many tens of miles and of some problems still to be solved in the design of a radar sonde system for measuring pressure, temperature and humidity profiles.

It is concluded that radar will become an established meteorological tool and some possible profitable future lines of research are indicated.

* Telecommunication Research Establishment.

## INTRODUCTION

It has been said that Fighter Command could not have won the Battle of Britain and that the weight of Bomber Command could not have been brought to bear on the enemy without radar. Radar has been hailed as the greatest single invention of the 1939-1945 War. When, in August 1945, it emerged from the cloud of secrecy in which it had operated throughout the war, attention was immediately turned to possible applications of radar in the field of civil flying. Radio waves have the advantage over visual and infra-red waves in that they penetrate cloud, fog, rain and snow with little or no loss. The use of them enables information to be obtained from a place or object which is obscured by weather or darkness and in the case of radar this information can sometimes be in the form of a picture.

In the application of radar to civil aviation the problem of deciding the type of information to send, the accuracy and detail required and the method of presentation of the information at the receiving end is a complex one. The one aim in all this work is to make flying safe, and in the present state of aviation this really means to make flying safe in bad weather. It is not the purpose of this paper to give an account of developments in the field of applications of radar to civil aviation, on which subject much has already been written. The work described here, however, has a common aim with this radar application in that from the aviation point of view it is hoped that it will lead to safer flying. The work is supplementary to the application work and seeks to use radar and radar techniques to help measure meteorological conditions and improve meteorological forecasts.

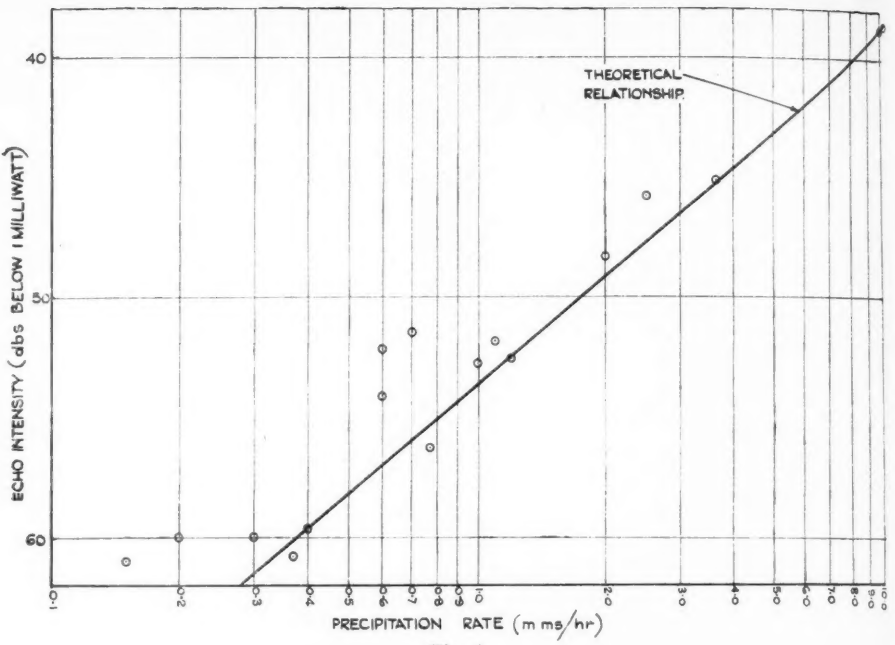



Fig. 1.
Radar echo intensity from rain at 3,000 ft. range as a function of precipitation rate.

## DETECTION OF PRECIPITATION BY RADAR

With the advent of microwave radar in 1941 it soon became apparent that echoes were obtained from precipitating clouds, rain and snow. During the war a limited investigation of this phenomenon was undertaken to determine its effect on the performance of radar sets. Recent research has been directed to discover the possibility of employing microwave radar to give advance warning of approaching rain belts. The work has brought to light some new evidence concerning the formation of rain, and it now seems possible that radar will not only be able to indicate distant areas of precipitation but will also be able to give a quantitative estimate of the amount of precipitation occurring.

### THEORY

The theory of radar echoes from rain, snow and hail has been worked out by Ryde.[1] He has shown that the echo intensity $P_r$ obtained from precipitation which completely fills the aerial beam is given by:

$$P_r = \frac{P_o G A N S (1 - \cos\theta) h}{4R^2}$$

where $P_o$ = power radiated by radar set.
$A$ = aperture of aerial (common transmitter and receiver).
G = gain of aerial.
$\theta$ = half angular aerial beam width.
$R$ = range of precipitation.
$N$ = number of scattering drops per unit volume.
$S$ = scattering function of each drop in the direction of aerial.
$h$ = pulse width.

For drops which are small compared with the wavelength, Ryde shows that the value of $NS$ is given by:—

$$NS = \frac{\Sigma N(D) D^6}{\lambda^4} f(m)$$

where $D$ = drop diameter.
$m$ = dielectric constant of material of the drop at the wavelength $\lambda$.

Rain, of course, comprises drops of various diameters ranging from .01 cm. to .8 cm. in very heavy rain. Some knowledge of the drop size distribution is therefore essential to evaluate the expression

$$\Sigma N(D) D^6$$

Best[2] has shown that determination of drop size distribution made in rain of the same intensity at different places may vary, but that on an average such distributions are substantially the same. In the analysis of the T.R.E. results the data correlating the average drop size distribution with precipitation rate compiled by Laws and Parsons[3] have been used.

## Experimental Work

The measurements at T.R.E. were made on a radar set working on a wavelength of 3.2 cm. directed vertically upwards towards the falling rain. Simultaneously measurements were made of the precipitation rate on a recording rain gauge. The correlation between echo intensity and rate of precipitation is shown in Fig. 1, together with the theoretical relationship obtained by using Ryde's formula with Laws' and Parsons' data for the average drop size distribution. Although measurements were made at various heights they were converted to equivalent values at 3,000 ft. on the assumption that, since the precipitating area completely filled the aerial beam, the power was inversely proportional to the square of the range. Measurements which were only made during conditions of steady rainfall confirmed that this assumption was valid and that precipitation was uniform from the ground up to the height at which the measurements were made.

## Bright-band Echo

When investigating echoes from rain with a radar set directed vertically, it was noticed that although the echo intensity decreased with height, as would be expected, a considerable increase in power was observed from a thin layer at a certain height. This strong echo, termed the "bright-band," is illustrated in Fig. 2. It has been observed by workers in many parts of the world. Experiments have established that the bright-band is located a little below the freezing level and that the echo power is 5—9 times greater than the echo from the rain just below the bright-band. Above that level the echoes are very much smaller and vary considerably in strength.

It has been suggested by Ryde[1] that in cases where the bright-band is observed, the particles at the highest levels consist of ice crystals which, because of the relatively low value of the dielectric constant of ice, give rise to a weak echo. As the particles fall they aggregate to form snowflakes, and these in turn melt to form raindrops during their passage through the freezing level. The dielectric constant of water is higher than that of ice and hence the radar signal becomes stronger. The raindrops will have a higher terminal velocity than the corresponding snowflakes from which the drops have formed. Thus, at some distance below the freezing level the concentration of drops will be less than in the region where the melting is taking place and the signal strength will fall. Measurements made at T.R.E.[4] on the ratio of the terminal velocity of a raindrop to that of a snowflake of equal mass, show that it has a value of the same order as the ratio of the echo power

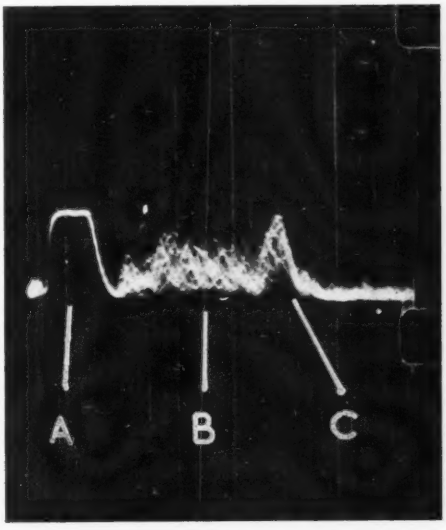



Fig. 2.
Bright-band echo.
**A** Ground transmitted pulse
**B** Rain signal
**C** Bright-band echo.

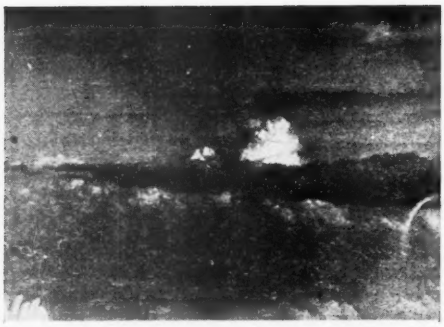


Fig. 3.

Cloud formation and corresponding radar echoes on plan position indicator display. Range 30 miles.

from the bright-band to that from the rain immediately below.

### Detection of cumulo-nimbus clouds

Within certain types of clouds such as cumulo - nimbus, convection currents are sometimes so violent that they present a danger to aircraft. Such clouds are invariably associated with heavy precipitation and it was therefore of interest to study the radar response obtained from them.

In the summer of 1946 a party of scientists from T.R.E. flew in a specially fitted Lancaster to the Singapore area to make an airborne study of the cumulo-nimbus cloud problem. The radar set operated on a wavelength of 3.2 cm. and a typical picture of cloud scene and corresponding echoes are shown in Fig. 3. It will be seen that the area of towering cumulo-nimbus clouds, the peaks of which extend higher than 30,000 ft., are clearly painted on the PPI picture. The performance of the radar set was such that these clouds were seen up to ranges of 100 miles, and measurement of the echo intensity indicated that the rainfall associated with them was of the order of two inches per hour, which is a normal value for tropical monsoon areas.

The Lancaster had been fitted with a recording accelerometer and it was hoped to fly into some of the cloud forms, but the pilot decided that the risk of flying into any area which "painted up" on the PPI tube was too great and thus there was no opportunity of obtaining any experimental correlation between cloud formations and turbulence.

### Practical applications of the radar studies of precipitation

The research work already undertaken has indicated that a wavelength of about 6 cm. is the optimum for rain cloud detection purposes. As no development work on this wavelength has been undertaken, as far as radar equipment is concerned, the choice has been made of using either 3 or 10 cm. for practical applications.

### Cloud warning equipment for aircraft

As a result of the experiments in the Singapore region a cloud warning radar for aircraft fitting has been designed at T.R.E. and subsequent development and production has been undertaken by E. K. Cole Ltd., Malmesbury. The apparatus works on a wavelength of 3.2 cm. and the performance is such that the position of heavy rain clouds up to a range of about 40 miles is indicated. A picture of the apparatus is shown in Fig. 4. Several sets are being installed in aircraft to enable extensive trials to be undertaken, and an assessment of the usefulness to aerial navigation is to be made.

### Ground stations for storm detection

With the greater power available it has proved profitable to use 10 cm. as the operating wavelength for radar ground stations to indicate storm centres. Such high performance equipment is capable of detecting heavy rain storms at ranges in excess of 150 miles and in tropical regions, the use of radar in giving early warning of the approach

of rain storms is proving invaluable, particularly to the forecaster who is concerned with the operation of aircraft in and out of airfields. It has proved possible to issue short-term forecasts of bad landing conditions, often to within five minutes, and to estimate the duration of the unserviceability of an airfield because of storm conditions.

## DETECTION OF CLOUDS BY RADAR —PULSED LIGHT

There is a requirement in meteorology, especially for use in weather ships, but also on airfields for a method of measuring the height of the cloud base from a single station.

The Ryde theory mentioned earlier indicates that the echo power from reflecting particles is inversely proportional to the fourth power of the wavelength. In June 1945, before the experimental work on the verification of the Ryde theory had been done, an opportunity was taken of observing clouds on an experimental radar set working on a wavelength of 6 mm. made at T.R.E. These observations showed that no echo was obtained from non-precipitating clouds even on such a short wavelength.

Now the average cloud is composed of water droplets of radii lying between 8 and 30 microns (one micron $= 10^{-6}$ metre), there being about 200 droplets per c.c. The Ryde theory indicates that the power of a set on a wavelength of 6 mm. with an aerial 30 in. diameter would have to be more than 300 kW. in order to give an echo from an average non-precipitating cloud at a range of 10,000 ft. Thus, the measurement of cloud height using radar on existing wavelengths is not generally possible and for this reason, attention has been given to the possibility of using the large peak power available in the visible light region of the spectrum obtained from electrical discharges in gases.

The experimental cloud base meter designed at T.R.E. uses as a source a high voltage electric spark produced between aluminium electrodes placed at the focus of a searchlight mirror. The present design of spark gap, which gives a pulse of light of less than one microsecond duration, is illustrated in Fig. 5. Further research is being undertaken on this unit to investigate the possibility of enclosing it in a pressurised envelope to increase the efficiency and to reduce the noise of the discharge.

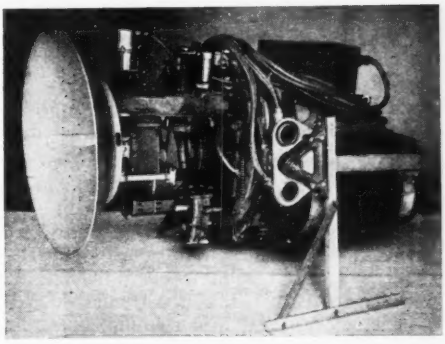


Fig. 4.
Cloud warning radar for installation in aircraft.

The pulse reflected from the cloud base is received on the second searchlight mirror which focuses the light on to a photo-electric cell, the resultant current pulse being amplified and displayed on the cathode-ray tube in the usual radar manner.

The peak power of the spark is about 11 megawatts, and the efficiency is such that with the mirror system used a flux of about four million lumens is obtained on a distant cloud. The performance of the original experimental apparatus was such that clouds up to 18,000 ft. were detected in daylight.

An engineered version of the set is being made and, in addition to the normal cathode-ray tube display, the set will incorporate a facility for recording on paper continuously at a remote point the height of the cloud base when it lies anywhere between a few hundred and 14,000 ft.

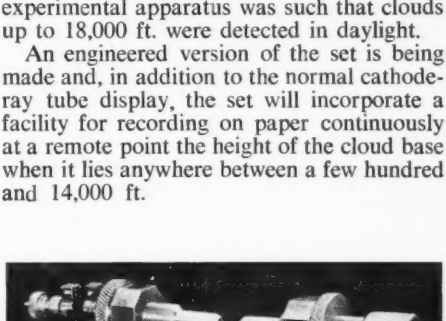


Fig. 5.
Spark transmitter for cloud-base meter.

## MEASUREMENT OF THE DENSITY OF THE UPPER ATMOSPHERE BY RADAR—PULSED VIOLET OR ULTRA-VIOLET LIGHT

Some meteorologists maintain that measurements in the upper atmosphere are at least as important as measurements in the troposphere. Much thought has been given to this study of the upper atmosphere, particularly with a view to determining how changes at such high levels influence the weather on the earth. Of particular interest is the study of ozone, water vapour and other gases which are known to exist at these high altitudes and which have absorption bands in the infra-red region of the spectrum.

The radiosonde equipment mentioned later in this paper enables routine measurements of pressure and temperature to be made up to altitudes of about 60,000 ft., but it is difficult to make measurements above this height. The variation in the total ozone content of the atmosphere concentrated at high altitudes has been studied by Dobson[5] who, using the sun as a source, measured one of the ozone absorption bands in the ultra-violet.

The Americans have recently published some measurements on the pressure and temperature profiles in the atmosphere up to a height of more than 60 miles, obtained during experiments with rockets of the V2 type. Such experiments will add tremendously to our knowledge during the next few years, but an application of radar suggested by the present author in 1946 provides a relatively simple method of exploring the upper atmosphere. The suggestion is an outcome of a proposal by Synge[6] who suggested measuring the molecular scattering from great heights by means of a modulated searchlight beam.

From Rayleigh's theory of molecular scattering it can be shown that the light scattered from unit volume of air is given by:

$$\sigma = \frac{32\pi^3\alpha^2 N}{3\lambda^4}$$

where $N$ = number of molecules per c.c.

$\lambda$ = wavelength of radiation.

$\alpha = \dfrac{\mu - 1}{N} = 1.08 \times 10^{-23}$ for air where

$\mu$ = refractive index.

As an approximation $\alpha$ can be considered independent of the scattering wavelengths and hence the expression reduces to:—

$$\sigma = \frac{330\alpha^2 N}{\lambda^4}$$

The measurement of the amount of light scattered back from a layer at any altitude will thus indicate the number of scattering molecules present.

The apparatus proposed for this measurement consists of a high power pulsed light apparatus, similar in arrangement to the cloud base meter already described. As the measurements will be made at night in a clear atmosphere a photo-multiplier can be used as a detector in place of the photo-electric cell. In addition, accurate range measurements are not required and hence the pulse shape is not as important as in the cloud base meter equipment. For this reason, also, wider pulses can be used and a discharge lamp can conveniently be employed as a source. This arrangement gives greater luminous efficiency than the system using a spark in air.

The cathode-ray tube display which should be obtained from such an experiment is indicated in Fig. 6. The length of the trace $XY$ represents 1,000 microseconds (100 miles). The length of the ground transmitter pulse $GR$ might conveniently be about 20 microseconds. The shaded area represents the energy received back from the scattering molecules although, of course, the purpose of the experiment is to measure the exact form of this shaded area. The receiver is provided with two gates or strobes, $A$ and $B$, which are adjustable in position (the width being half that of the transmitter pulse). The energy received in each of these strobes is integrated separately and compared in an electrical balance type of circuit. Thus, the energy scattered by an illuminated layer of one mile in depth (10 microseconds) at height $a$ can be compared with the energy scattered by a similar layer at height $b$. For absolute signal to noise ratio measurement it will be necessary to move the strobe $b$ along to the end of the trace where no appreciable energy is received. At extreme ranges very little signal will be received but measurements will be possible with long integration times, since circuits have been designed which integrate satisfactorily up to a period of several hours.

Experiments on the pulsed light system of exploring the upper atmosphere are being

made at the Ministry of Supply Radar Research and Development Establishment at Malvern, in conjunction with the Meteorological Office.

## RADAR FOR MEASURING WIND

The importance of accurate wind finding in aviation has been appreciated from the earliest days of flying. With the general tendency for aircraft to operate at higher altitudes the problem of providing accurate wind data has become more difficult, but once more radar can come to the aid of an aviator.

A radar system of wind finding has been in use in Great Britain for some time. The measurement is made by following a balloon-borne radar target, taking the form of a corner reflector, from a powerful 10 cm. ground radar station. Limitations in this system, which provides a big advance over anything available previously, are that the range obtainable on the reflector is limited to about 30 miles under the best conditions, the accuracy is dependent on the skill of an operator at the ground station, who turns the aerial to follow the target and measurements of average wind during one minute or more only are possible.

The problem of long-range accurate wind finding has been studied at T.R.E. and a system employing secondary radar, *i.e.* a pulsed ground station working in conjunction with a responder, has been designed and is in process of development.

In this system the outgoing transmission from the ground consists of two microseconds pulses on a wavelength of about two metres. The associated balloon-borne responder consists of a miniature two metre receiver, the output of which is fed via a modulator valve to a small triode oscillator working on a wavelength of about 10 cm. The power supply of the balloon-borne system consists of small batteries which have been specially developed for the purpose. Thus, the outgoing two microsecond pulses from the ground station are transmitted back on a wavelength of 10 cm. and these are received on a paraboloidal aerial 5 ft. in diameter. The aerial system has been designed to "lock on" to the responder signal and to "follow it" during its motion through the atmosphere quite automatically and with great accuracy. The "lock-follow" aerial system indicates the azimuthal angle $\theta$, the elevation angle $\phi$ and in addition the radar display indicates the range $r$. The differentials $\dot{\theta}$, $\dot{\phi}$ and $\dot{r}$ are also available and it is intended to feed these parameters into a computer which will indicate values of the velocity and direction of motion of the responder, *i.e.* wind speed and direction, on two dials.

The experimental responder is illustrated in Fig. 7. The apparatus is completely

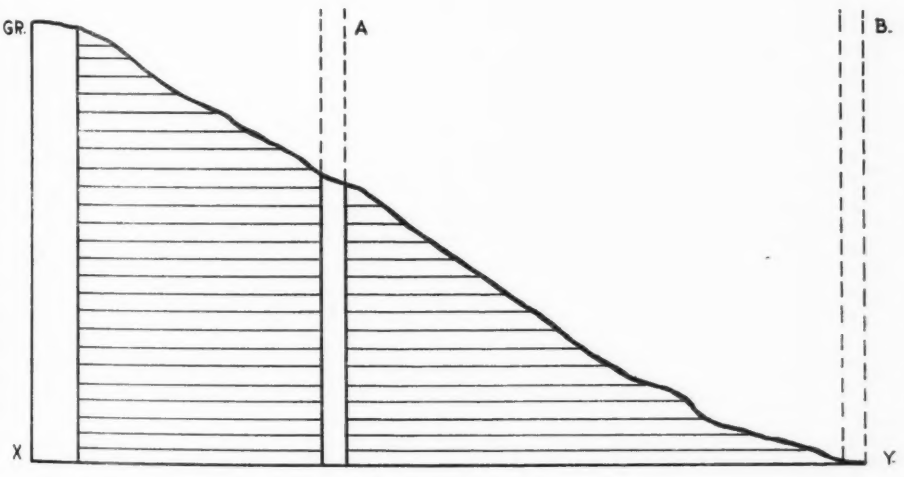


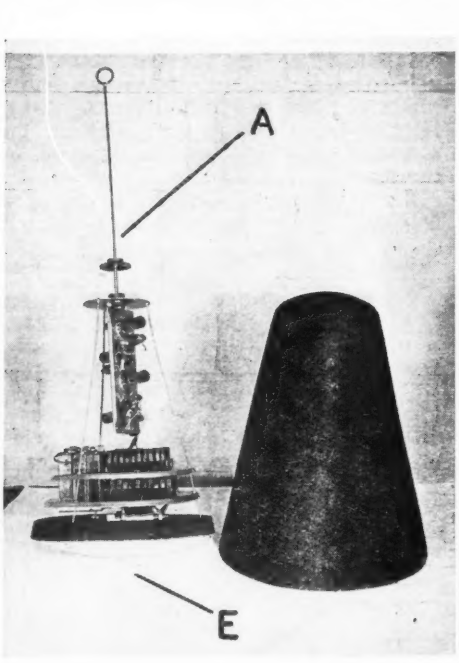

Fig. 6.
Radar display for measurement of molecular scattering.

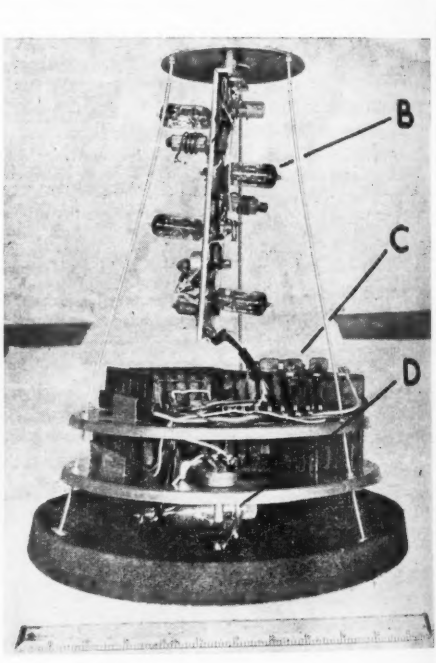
housed in a thermally insulated container made of expanded rubber. The efficiency of this container is such that the heat dissipated by the battery and electrical circuits maintains the internal temperature constant at about $+10°C.$, even when the external temperature is kept at $-50°C.$ for 90 minutes. The "lock-follow" aerial system which forms one of the main features of the ground station is illustrated in Fig. 8. It is the same aerial as is used in the Army gun-laying radar apparatus, as might well be expected when it is remembered that the problem of wind finding in this system is identical with the problem of accurate location of aircraft in normal radar.

Early flight trials with the wind finding system have indicated that the responder can be followed easily to ranges of 100 miles. The maximum height reached during any ascent has been 70,000 ft., the limitation being that the balloons have burst at this height. There is little doubt that the system will provide the most accurate wind finding method yet available, and experiments will be undertaken to assess its usefulness for the study of gusts.

## MICROWAVE RADAR SONDE

The most widely used meteorological instrument is the radiosonde. The sonde system in general use in this country and overseas consists of a radio apparatus operating on a wavelength of about 11 metres, which is arranged to transmit readings from the balloon-borne meteorological instruments. Wind speed and direction can be obtained from this system, although not very accurately, by measuring the bearings of the balloon-borne transmitter from three suitably located ground stations. During the past two or three years corner reflectors have been




Wind finding responder.

Fig. 7.

A Receiver aerial
B Receiver
C Batteries

Fig. 7.

D Centimetric transmitter
E Centimetre aerial.

attached to the balloon in addition to the radiosonde, and the wind measured by radar as previously mentioned.

At T.R.E. the wind-finding responder is being adapted for use as a "radarsonde," meteorological instruments to measure pressure, temperature and humidity being mounted towards the top of the responder. For the present these instruments are of the conventional type taking the form of an aneroid capsule for pressure, a bi-metallic strip for temperature and a piece of gold-beater's skin for humidity measurement. The fundamental problem in telemetering the information from the instruments is to transmit to the ground with extreme accuracy the angular displacement of the arms on these instruments.

The actual radar pulses are used as a communication channel in the system and various methods of achieving the high degree of telemetering accuracy required are being tried. The first experimental model used a chronometric unit and in this system the angular deviation of the arms of the instruments is measured in units of time. The units of time and the integral number of these units pertaining to each instrument is then transmitted to the ground station by means of a width modulation of the radar pulses sent out from the centimetre triode valve.

The experimental chronometric unit assembly which has been made by Kelvin, Bottomley and Baird Ltd., Basingstoke, is illustrated in Fig. 9. The small electric motor is geared to a cylinder on which is wound a helical ridge. This ridge is made to contact a reference arm *A* and later arms *B*, *C* and *D* of the three meteorological instruments. It is arranged that the motor makes a mark in the form of a few wide pulses on the pulse channel each time it completes one revolution, and these are the time marks referred to above. The reference arm *A* is arranged to give a short double burst of very wide pulses and each meteorological arm, a single burst of pulses of similar width. Thus, at the ground station the reading of the instruments is obtained by counting the number of timing pulses, that is the number of revolutions the small motor makes, between the double reference mark due to the reference contact *A* and the meteorological arms *B*, *C* and *D* respectively. The gear ratio between the motor and the helical wound drum is 500, so that the position of this drum is telemetered

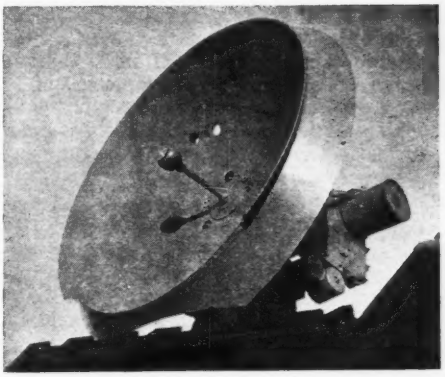


Fig. 8.
Receiver aerial for wind finding and radarsonde ground stations.

to one part in 500. The speed is such that one revolution of the drum takes approximately 15 seconds, so that one reading of each instrument is obtained in this time interval, that is about every 250 ft. of ascent.

It has proved difficult to achieve the necessary high degree of accuracy required by the meteorologists—one part in 1,000—with the mechanical moving parts in the chronometric unit and the possibility of making an all-electronic unit is being investigated. At the ground station the readings of the instruments will be recorded automatically, both on graph paper and a

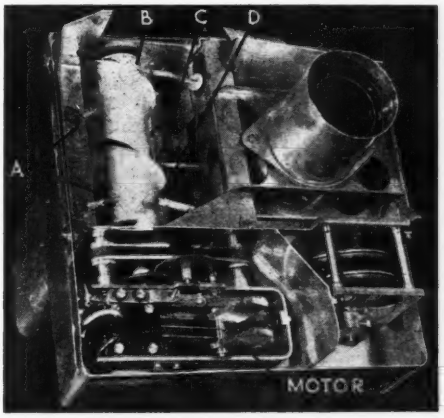



Fig. 9.
Chronometric unit for radarsonde.

teleprinter, so that they can be passed easily to the forecaster.

Considerable experimental work has still to be done in the development of a meteorological radarsonde, but the work done so far indicates that microwave radar will play an important part in the future of this very important instrument.

## CONCLUSIONS

A brief outline of the work at T.R.E. on the application of radar to the science of meteorology has been given. Much remains to be done in this initial work and there are many problems on which work has not yet started. Examples are the measurement of turbulence in precipitating clouds, using radar sets equipped with apparatus for measuring the Doppler frequencies produced by the moving raindrops, the use of powerful radar sets to study atmospheric regions which apparently give rise to echoes in the absence of cloud, and the use of the microwave balloon-borne responder to investigate gusts. There seems little doubt that radar is becoming an established meteorological tool and that it will play an ever-increasingly important role in that science in the future.

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# DISCUSSION

**Sir Robert Watson-Watt** (President of the Royal Meteorological Society, *Fellow*): The first major trouble in the development of the earlier and comparatively low frequency radar systems was the appearance of echoes of substantial magnitude from apparently clearly marked tropospheric discontinuities. Could Dr. Jones say whether they had reached some sort of cross-connection between the microwave observations of to-day and the comparatively crude observations in the 20 megacycle band on the co-called "z-layers"—a term then used to cover their ignorance!

They passed then to the problem of radar echoes received from objects at exceptionally long ranges, the phenomenon which had been studied in great detail as "unorthodox radar visibility." Through the remarkable work of Dr. Henry Booker, in particular, these studies had been brought to a stage at which the radar man was waiting for his meteorological colleagues to provide him with finely detailed information on the temperature and humidity distribution in the lowest few hundred metres of the atmosphere from hour to hour, so that the theory of radio propagation in the troposphere might be tested and further developed, to the advantage of meteorology and radiophysics alike.

From there they had gone to the earliest proposals for the application of primary radar against an artificial balloon-borne target as an implement in wind finding. Next they began to find themselves competing with an ever-growing complexity of precipitation echoes of the kind which had formed the subject matter of a substantial part of Dr. Jones' paper. Those precipitation echoes had at least three important aspects to the technical user. They had been concerned first in deciding how they could mitigate their objectionable effects in masking and comprising the desired features in the radar P.P.I. picture. Secondly, they had examined how air-borne radar sets might give facilities, at a fairly stiff price in bulk and weight, for pathfinding among cumulo-nimbus clouds; it was fortunate that cumulo-nimbus clouds, although they occurred in packs, were not densely packed, there being relatively quiet channels between the turbulent clouds; channels which could be traced from afar, and closely navigated, by radar. The third aspect was the fruitful and stimulating one which Dr. Jones had so ably described, the use of radar as an aid to the study of the atmosphere.

From these first openings of avenues still under exploration, the radar man passed to the development of microwave technique, and one of the many elegant applications of that technique lay in the secondary radarsonde which had been demonstrated. He

recalled a demonstration of the use of pulsed light in the study of the atmosphere he had seen in the Autumn of 1946, at the National Radio Electric Laboratory in Paris. What had impressed him on that occasion was the tenuous nature of a haze layer, quite indistinguishable to the unaided eye, which was revealed and located by the pulsed light beam technique. The tenuity of that particular target led him to share Dr. Jones' optimistic view about the possibility of usefully sounding the much more tenuous areas of the much higher atmosphere.

He hoped that during the Discussion they would hear something about the next steps in connection with that group of elegant and fascinating projects. What was the priority list in those possible applications of the technique? They must continue to ask earnestly for a priority list, because his second question was " What man-power and money-power were now available for such work? " he was sure the immediately discouraging answer would encourage them to request that the priority list should be considered most carefully, so that the best return could be obtained from the work of their too limited group of extraordinarily ingenious and able investigators.

**Dr. F. J. Scrase** (Meteorological Office): He was directly concerned with the development of meteorological equipment, and had kept in close touch with Dr. Jones in those applications of radar techniques. The greatest advances in practical meteorology during the past decade had been brought about by the application of radio and radar. Sir Robert Watson-Watt had asked if the meteorologists had indicated their priorities for the various applications. They had been stated officially; but there were one or two of the more urgent needs which should be met.

The application of radar to the detection of precipitation was not only of interest as a means of locating storms, but also as a possible method of estimating the drop size and liquid water content in the atmosphere. Those were vital factors affecting ice accretion on aircraft, and it was difficult to obtain sufficient information about them by measurements with instruments carried in aircraft.

Another urgent need which radar might be able to meet was the investigation of the characteristics of gusts in dangerous clouds. In clear weather, one way of doing that was by visual or photographic observations of the effects of gusts on smoke trails. If trails of radar-reflecting material could be projected into turbulent parts of the cloud it should be possible to follow the movements and break-up of the trails by radar.

The greatest need was for more observations of upper winds and temperatures. Much information about them was already available up to heights of 50,000 ft. in certain parts of the world, but aviation developments now required extension of the data to 70,000 ft. over the whole globe. Improved radar systems seemed likely to meet the height requirement, but unless the equipment became less expensive, not only in initial cost but also in maintenance, it was unlikely to be adopted very widely by some of the smaller meteorological services. On the meteorological side they were endeavouring to keep ahead of that need for extending the data to 70,000 ft. by improving the performance of sounding balloons so as to be able to reach 100,000 ft. The stage had been reached of being able to obtain ascents up to 90,000 ft. fairly readily.

**N, E. Rowe** (British European Airways, *Fellow*): They needed accuracy of forecasting of the wind in time, in place, in its direction and its force. As an example of its significance with a medium-size aircraft of, say, 150,000 lb., on a 10-hour flight, a difference of 5 m.p.h. between the forecast and the actual wind strength would make a difference in the payload of about 5 per cent. The accuracy of forecasting wind was of the greatest significance in planning routes and the loads which could be carried on those routes, and hence the operational economy.

He believed there was some theoretical support for the view that wind sheer, *i.e.* the rate of change of wind with height, might be a significant parameter in giving rise to the gusts which were experienced in clear air. What sort of accuracy could Dr. Jones ensure concerning wind speed with height? What they were looking for was an accuracy of perhaps 2-4 m.p.h. change of wind speed, or maybe a little more, in 1,000 ft. at levels of 20,000-40,000 ft., and perhaps higher.

How far was it possible, using the new technique, to get the information from the air that was approaching, rather than from the air which was receding? That was a matter of getting aircraft out to drop the balloons rather than sending them out from the ground.

Was there any application of the techniques discussed to the determination of thicknesses of fog and possibly slant visibility through fog, radiation fog in particular? He had been impressed by the similarity of the formulæ in the paper to those which applied to the attenuation of light in low visibility.

**Gordon Hughes** (Henry Hughes & Son Ltd., *Associate*): He would like more information about the presentation of conditions that was given to the pilot of an aircraft. How could the pilot tell how bad the conditions were? When the pilot at Singapore had preferred not to go into the clouds, was it because he could not see what the conditions were, or was it because he did not like the presentation on the screen? There must be a standard by which a pilot could tell when a storm was a bad one and when conditions were too dangerous.

It had been suggested that a set would be made by which to determine the height of the cloud base; but that would mean adding another set and having someone else to watch it. Would it be possible to devise one set which would do all the things required?

What were the accuracies attained with the old system and how did they compare with those attained with the new system?

When all the work was done and accuracy of wind finding was achieved, was there a chance of devising some means of getting the information to the forecasters in good time, so that they could use it to advantage? It was constantly said that, although information was made available, it reached the forecasters two or three hours later.

**Squadron-Leader E. Coton** (Royal Aircraft Establishment, *Assoc. Fellow*): At the time the pilot at Singapore had decided against going into cumulo-nimbus clouds, he had had every reason to believe that, if he went through one of those clouds in a Lancaster or, in fact, in any heavy bomber which was not specially strengthened, the machine would break up. That picture had since been changed by the extra knowledge gained, and that extra knowledge was due entirely to the investigations made possible by radar techniques. But at the time the pilot concerned decided against going into the clouds there was practically no information available, and what little there was clearly indicated that the danger was great. Thus, he felt that the pilot was entirely justified.

As a pilot he thought it was an understatement to describe radar merely as an aid to the study of the atmosphere; it was far more than that; the investigation of cumulo-nimbus clouds was just not possible without it. They had never had anything before which did the job so well as the cloud base indicator. The reports of cloud bases over aerodromes were in the past very often not much more than guess-work; but the cloud base meter would give the honest truth, he believed, which was something they had never had before in that connection.

**A. C. Clarke:** Some time ago an American scientist had reported in the *Proceedings of the Institution of Radio Engineers* that unexplained transient radar echoes had been detected in a perfectly clear sky, and the picturesque name of "angels" had been given to them. Had any further results been obtained in that field?

He believed that Russian experiments with searchlight technique had given readings up to 35 miles; what information was available on that work? He did not think it was a pulse technique, but simply a matter of observing diffused light.

Bearing in mind the extreme sensitivity of optical systems and detectors, it seemed that it would be possible to use the optical type of radar for astronomical work, *e.g.* for "ranging" the moon. Had anything been done on those lines?

When he had first studied storm clouds on high-resolution radar he had been impressed by their extremely complicated structure, and in particular by the way in which parallel bands of rain seemed to be formed. He had also seen some very fine extracts from a film of a typhoon observed on a P.P.I. screen in which the circular formations round the "eye" could be watched. Had that film reached Great Britain yet, for it should be of great interest.

**Flight-Lieut. P. M. S. Hedgeland:** Dr. Jones had stated that no echoes had been received from non-precipitating clouds, but he had observed such an echo. He had been ground-testing a 3 cm. radar equipment in the Suez Canal Zone where it was possible to receive an echo from a range of hills about eight miles distant. He was surprised to see a very bright response of about two square miles in area occurring between the aircraft and the hills where normally there was no

echo. On getting out of the aircraft he had noticed a large isolated cloud and there was definitely no precipitation occurring between him and the hills, the visibility being crystal clear.

He referred to some observations of the responses from cumulo-nimbus clouds when in flight; on one occasion the aircraft, a Lincoln Mk. B.2, had flown at 15,000 ft. through a small cloud giving such a response. There was no accelerometer on board, but the bumping was extremely violent and most unpleasant; he would not care to do it again.

**E. Gold** (Royal Meteorological Society): Many people who were not meteorologists assumed that the atmosphere remained, broadly speaking, regular and constant in space and time; but in fact it did not remain regular and constant in space or in time.

There were two kinds of wind which were important in aviation. One was the kind which would affect a flight of several hundreds of miles, and in that case the meteorologist was in a position to give in advance information with a fair degree of accuracy; the irregularities, broadly speaking, cancelled one another. On the other hand, there was the wind which somebody wanted for a particular place at a particular time; at a slightly different time it would be different, because the atmosphere did not behave in a regular way.

The comments made concerning the pilot at Singapore who would not fly into cumulo-nimbus clouds had reminded him of an experience of a meteorologist flying in a two-seater aircraft who asked the pilot to fly into a cumulo-nimbus cloud, in order to see what happened. The aircraft was knocked about so much that he thought he would never get out alive. It was understandable, therefore, that the pilot of a Lancaster would not wish to fly into such a cloud, especially in the Singapore region where it was likely to be of great intensity.

Communication systems were of great importance. It was well known that during the war much information available from the radar instruments could not possibly be communicated and distributed sufficiently rapidly to enable it to be used; there was always some prior operational objection or operational use of the lines which had prevented the rapid distribution of the information to the people concerned.

Dr. Jones had made no reference to the American cloud height indicator; did that indicator use a method similar to the methods described in the paper? It had been in use during the war and since, and its great advantage was that it indicated cloud height instantaneously.

In Paris, in 1945, Dr. Bureau had been working on a daylight cloud height indicator in which he had used the visible part of the spectrum; he believed also that Dr. Bureau had used the carbon arc. Did Dr. Jones know anything about that work?

Why was the cover of the wind-finding responder black? If the heat inside were sufficient to maintain it at a reasonable temperature during the night, it might well attain a very unreasonable temperature during daytime, in spite of the low thermal conductivity of the covering material. On the other hand, if the cover were white, it would not interfere with the efficiency of the instrument at night and would give a lower internal temperature during the day.

With regard to the echo intensity, $P_r$, given by a formula with $\lambda^2$ in the denominator; if $\lambda^2$ were right and if $\lambda^4$ were right in the formula for *NS*, one would expect the echo to vary inversely as $\lambda^6$, whereas it varied inversely as $\lambda^4$. He had no doubt there was a satisfactory explanation, but it would be worth while to make it clear in the paper.

**N. J. Hancock** (*Assoc. Fellow*): He did not quite appreciate the difference between a responder and a transponder; it seemed that the instruments were practically the same.

Was it a fact that, the shorter the wavelength that was used as a probe, the greater was the possibility of masking or blanketing by dense local fog or cloud in the region of the instrument?

He asked for some degree of uniformity in the denominations used. For example, it was difficult to follow when terms of kilometres were mentioned one minute, miles the next and thousands of feet the next minute. It was equally difficult to appreciate comparisons when megacycles, microns, centimetres and so on, followed in quick succession.

**C. E. G. Bailey:** In the paper it was stated that the degree of accuracy required by the meteorologists in respect of the variables other than the chronometric variables, was one part in 1,000—in other words over the range of variables of the order of 0.1°C in temperature, 0.1 per cent. in humidity and somewhat less than one millibar. Did useful

meteorological information really demand accuracy of that order?

There was one set of variables which the layman could very well understand would play an exceedingly important part in the economics of aircraft flight, and that was the chronometric set, the wind velocities. Another contribution to the solution of the meteorologists' problems might be made by the workers in the field of electronics. When scattered data were collected about wind speeds at various heights, they could be presented to a pilot or navigator, who could then judge which course he would follow; it was clear that the economics of an individual flight might depend very largely on his good judgment as well as on the "luck" of the meteorologist. That sort of data, once it was properly boiled down, was very suitable for offering to a computing machine. It was not altogether fantastic to suggest that the available calculating machines should be devoted for a certain number of minutes each day to the routine solution of problems of this kind.

**Gordon Hughes:** Dr. Jones and his fellow scientists had known what to expect in those clouds, and pilots were also prepared to face the same risks. He thought British pilots would not forgive the Ministry of Supply for having used automatic pilots in supersonic flights instead of human ones, thus allowing the Americans to be first at flying faster than sound. There were enough pilots in Great Britain who would welcome the chance of doing anything like that.

**S. Scott Hall** (P.D.T.D.(A.), Ministry of Supply, *Fellow*): Criticisms had been made of the bulk and weight of some of the radar equipment, but they ought to take into account the structural weight which might be saved in very large aircraft by its use, provided that it were 100 per cent. reliable and would always tell the pilot how he could avoid the gusts which might break up his aircraft. Mr. A. E. Russell, when discussing before the Society the design of the Brabazon I, said that the incorporation of his gust alleviator had resulted in a saving of something like 16,000 lb. in structural weight. The equipment mentioned by Dr. Jones was an alternative to the gust alleviator and would save this weight in an aircraft of the size of the Brabazon I if their knowledge of gusts were correct. Looked at in this light the weight of the equipment itself was not so serious.

It was most important that there should be a direct correlation between the intensity of gusts and their indication on the tube.

## DR. JONES' REPLY

**Sir Robert Watson-Watt:** Sir Robert had referred to the echoes he had observed on 10 metre radar in 1936. One night at a radar station on the East Coast in 1940, soon after he had joined the R.D.F. party, he and his colleagues had "watched" what they thought was a very large formation of German aircraft going up the North Sea towards Scapa Flow where much of the Fleet was. There was a great anxiety when "watching" the aircraft going up the coast at a range of something like 100 miles. When 20 or 30 miles from Scapa, the aircraft had turned round, and gone back again. He did not think the matter had ever become quite clear, but it seemed possible that it was one of those peculiar echoes. There seemed no doubt that they did experience peculiar echoes on those short wavelengths, and it was unfortunate that nowadays 10 metre radar seemed to have died a natural death, so that there were few chances to make experimental observations.

Under certain conditions they appeared to obtain echoes when there was nothing at all to be seen in the atmosphere. The Americans had reported such echoes when making observations with very high powered radar sets. If they could get a very powerful radar set, say megawatts of power, on 10 cms., it would be interesting to look for what there was to be seen in the upper atmosphere.

**Dr. Scrase:** Dr. Scrase had replied to Sir Robert's point concerning the priorities of the meteorologist.

**Mr. Rowe:** The new radar wind finding system was designed to give a vector accuracy of 2½ knots. The actual answer was obtained by feeding the information as to the position of the balloon into the computer continuously. It was hoped to achieve the vector accuracy of 2½ knots by integrating this information for as short a period as 10 seconds, that is, the wind information would be given "continuously" but any reading would indicate the average wind experienced during the previous 10 seconds of flight.

The Meteorological Office were actively engaged on a method of measuring from the ground the actual visibility between an air-

craft and the ground. They had a method involving the use of visible light but it employed a sinusoidal rather than a pulse modulated system. It was hoped to determine by measurements from the ground, how far a pilot could see under fog conditions when coming in to land, so that he could be given accurate visibility data before his final approach.

**Mr. Hughes:** He could only stress that at the moment they lacked evidence of correlation between radar echo and gust. The term "cumulo-nimbus cloud" covered a multitude of sins and many pilots reported that they had flown through them and had felt nothing at all. On other occasions they knew that aircraft had flown into such clouds and had been broken up.

An experimental cloud warning set had been tried in a Transport Command aircraft in the Far East and the pilot and crew had given glowing reports of it. He felt, however, that all that came out of it at the moment was that if an area "painted up" on the cathode ray tube then it was desirable to fly round it if possible. They might at some time in the future, perhaps by scanning in the vertical plane, and hence getting some idea of the extent in height of the cloud, be able to get some idea of the degree of danger it represented, but such information was lacking at the moment. It should be stressed that the work and methods he had mentioned were yet in an early stage of development.

It was planned to put the cloud-base meter at some convenient point on the airfield and to provide the cloud height in the form of a continuous recording on paper at some distant point—for example in the control tower. If the meteorologist found it desirable, there was no reason why the readings of the wind finding computer and the radarsonde should not be sent directly over the teleprinter system to any remote point, for the system of presentation was ideally suited to this purpose.

**Mr. Clarke:** He had already referred to the phenomena of "angels" in the conclusion to his paper but very little was yet known about them. With regard to the Russian searchlight experiments they had installed a very large number of searchlights to put up a concentration of light in a particular area of the sky and had measured the molecular scattering by employing sinusoidal modulation. They believed, however, that pulse was a much better method.

Longer wavelengths than visible light were needed in order to get an echo from the moon. He had started building a radar set to get a moon echo in 1945 but had abandoned the idea when the Americans published their success towards the end of that year.

**Flight Lieutenant Hedgeland:** It was an interesting point about the occurrence of an echo in the absence of rain. Although he and his colleagues at T.R.E. had never observed such an echo they had seen echoes when no rain was recorded on the ground. The radar set, however, clearly showed rain leaving the cloud but it had dried up before reaching the ground. On the other hand, it was almost certain that if they came close enough to a well defined cloud with a radar set they would get an echo, but for long ranges it would be of the wrong order of magnitude.

**Mr. Gold:** He had heard of both the American and French cloud height meters. He understood the Meteorological Office had been trying to get one of the American equipments for some time, but that there had been considerable teething troubles experienced with the design, and that no delivery date had yet been forthcoming. The cover of the wind finding responder was of such a thickness that solar radiation had no apparent effect. He agreed that it appeared desirable to adopt Mr. Gold's suggestion and paint it white. The comment on the $\lambda^2$ and $\lambda^4$ terms was a good one, and the explanation meant bringing into the argument something more about the constants of the radar set. He would try to present the equation more clearly in the final version of the paper.

**Mr. Hancock:** The word "transponder" was an American term, while the word "responder" had been used even by the early workers on R.D.F. in this country. The Americans now used both words, defining a transponder as a pulse repeater which received and transmitted on different wavelengths, while a responder received and transmitted on the same wavelength. If they adopted the American nomenclature then they would refer to the radarsonde as a "transponder" rather than a "responder."

It was certainly true that the shorter the wavelength used as a probe, the greater was the attenuation due to fog. It was well known that neither infra-red nor visible light would penetrate dense fog, but because of the $1/\lambda^4$ law, the attenuation of radio wavelengths by fog was extremely small.

He also was unable to clear up the worry over units. Radar ranges were nearly always measured in miles, height of aircraft in feet, radio transmissions in megacycles or metres, while in the optical region it was usual to refer to a "micron" rather than "one millionth of a metre."

**Mr. Bailey:** He was sure that meteorologists knew that the present meteorological instruments were certainly not accurate to one part in a thousand. It was hoped, however, that the present instruments did not represent the last word. In the design of the radarsonde T.R.E. were asked to make provision for better instruments when they came along, the final aim being to measure the meteorological condition to the high accuracy demanded by the forecasters. The problem of providing such accuracy meant using a telemetering system which was accurate to one part in 1,000. He believed that they had broken the back of the problem by arranging that the circuits were maintained at a constant temperature during flight, for the difficulties of providing an accurate telemetering system in a device in which the temperature changed from about $+20°C$ to $-60°C$ were obvious. He believed that there was a reasonable hope of achieving the accuracy required for the sonde, especially if they could replace the present instruments, which measured the meteorological condition as a function of a mechanical displacement, by ones which gave indications in terms of an electrical unit direct—for example a thermistor bead for measurement of temperature.

**Mr. Scott Hall:** The radarsonde could be used for gust investigation in one of two ways. First, an accelerometer could be carried in the responder and the gust information so measured could be telemetered over the channel provided. Secondly, it was possible to measure range extremely accurately with radar, and if the flight of the normal wind finding responder were followed from two suitably placed ground stations, an analysis of the range measurement from each ground station, recorded say, on ciné film, would give accurate information on gust.

# THE DESIGN AND DEVELOPMENT OF THE HELICOPTER ROTOR TESTING TOWER

by

R. F. TAYLER, A.M.I.Mech.E.

## 1. INTRODUCTION

From the start of the development of helicopters at the Bristol Aeroplane Company the necessity was appreciated of providing means of testing, independently of the aircraft, rotor hubs and blades. The usual method of testing on the helicopter suffers from several disadvantages, the most important being that proximity of the ground beneath the rotor greatly affects the flow through it and makes such performance figures as can be obtained unrepresentative. Others are the probable inconvenience of having to wait for the completion of the aircraft before rotor-testing can start, the comparative difficulty of instrumentation, and the risk of damage to the aircraft by failure or misbehaviour of the rotor.

Consequently, a proposal was made to the Ministry of Supply that a testing tower for rotors should be built and that it should have the following general characteristics:—

(1) A height sufficient to allow the largest rotor contemplated to operate above its "ground cushion."

(2) Accurate means of measuring, and recording the power absorbed, the lift, the rotor control forces, and other quantities as necessary.

(3) A source of power capable of driving all sizes of rotor envisaged for long periods at speeds greater than their normal operational speeds, and of developing its full power at low speeds.

(4) The upper section designed in such a way that, with a minimum of modification, it could accommodate configurations other than that of the single rotor. Such configurations might include co-axial rotors, inter-meshing rotors with small offsets and rotating on axes having a mutual angular disposition, or the mounting of a complete helicopter to study the effect of rotor downwash on the fuselage surfaces.

This proposal was approved by the Ministry of Supply, and a contract for the design and erection of the tower was issued.

## 2. SPECIFICATION

The following are the limiting conditions for which the tower has been designed.

(1) Vertical distance between ground and centre of rotor is 50 ft.

(2) The maximum rotor diameter is 70 ft.

(3) The speed range is from 50-700 r.p.m., depending on rotor size.

(4) The maximum permissible rotor lift is 15,000 lb.

(5) The torque available at the output shaft is 250,000 lb. inches. The full torque is developed at a speed of 180 r.p.m., this being the speed at which a 70 ft. diameter rotor is estimated to absorb its maximum power.

(6) The power associated with the limiting conditions of (5) is approximately 675 h.p.

(7) The maximum permissible out-of-balance force at the rotor hub in any horizontal direction is 100,000 lb.

Paper received January 1949. Mr. Tayler is with the Bristol Aeroplane Co. Ltd.

2.1. The following measuring and recording means are at present available.

(1) A lift or thrust indicator.
(2) A speed indicator.
(3) An ammeter and a voltmeter for power measurement.
(4) Indicators for showing, in two mutually perpendicular directions, the horizontal out-of-balance forces at the hub.
(5) Three indicators showing the forces applied to the pitch controls by each blade.
(6) An indicator showing the position of the rotor in azimuth at any instant.
(7) An indicator showing the collective pitch setting, and means of varying this setting during rotation.
(8) A continuous film camera to record those indications which fluctuate too rapidly to be read visually.
(9) Optical equipment for observing the blade tip paths for purposes of "tracking" correction.

The instrumentation is described in more detail in Section 4 and in the Appendix.

## 3. GENERAL DESCRIPTION

General design of the tower is shown in Figs. 1, 2 and 3. The structure is best described in its individual sections.

*Foundations*

The foundations take the form of a reinforced concrete platform, hexagonal in plan, with six legs running radially outwards. The whole has a depth of eight feet, two feet of which is below ground.

*Pylon structure*

This is a fabricated, four-legged steel structure which straddles the power unit and supports the tower proper. Its feet are embedded in the foundation platform, and it terminates in a spherical seating in which rests the lower end of the tower body.

*The tower body*

Designed as a column, the tower body is about three feet in diameter and 25 ft. in height. It is constructed as a built-up steel tube having internal longitudinal stiffeners and occasional transverse diaphragms. The centre diaphragn contains a bearing which serves to steady and to support the weight of the driving shaft.

The tower body is pivoted at its base in the spherical seating on the pylon structure, to which it is held by a ring of spring-loaded bolts, and supported by six twin guy-ropes. Each of these ropes is attached at both ends to the upper end of the tower body, forming a loop which passes over a large diameter half-pulley mounted at the outer end of each of the foundation legs. The height of the half-pulleys may be adjusted, varying the tension in the ropes and hence the natural frequency of the system. Vibration of the cables is restricted to the fundamental mode by restraining them with light but stiff frames, which in effect, transform the cables into linkages (see Fig. 1).

The minimum frequency of the tower is about 1.0 c.p.s., the condition approximating most nearly to free flight and in which most running is done. Higher frequencies may be obtained to simulate undercarriage resonance or other conditions. The greatest amplitude of lateral movement at the hub is about 12 in.

*The upper structure*

As designed for the single-rotor configuration, this is conical in form, some eight feet in height, and is rigidly bolted at its base to the top of the tower body. An access and observation platform is fixed to the cone about four feet below the rotor.

This cone contains the final drive shaft, known as the rotor axle. This is distinct from the main drive shaft which terminates at the top of the tower body, and the two are connected by a universal joint which can transmit torque but no end-thrust. The axle may therefore be considered to be axially free of the main shaft, and when stationary or running under conditions of zero or negative lift, the weight of the axle and rotor is taken by a ball thrust race. When, however, the lift exceeds the weight of the axle and rotor, the upward load is carried by a taper-roller thrust race mounted in the lift-measurement device which is described later.

The axle is held radially by plain bearings at top and bottom. The upper one is held by the units measuring out-of-balance forces, but should these forces be so large as to overcome the measuring units the axle can move laterally only as far as the wall of the clearance hole by which it emerges from the top plate of the upper structure.

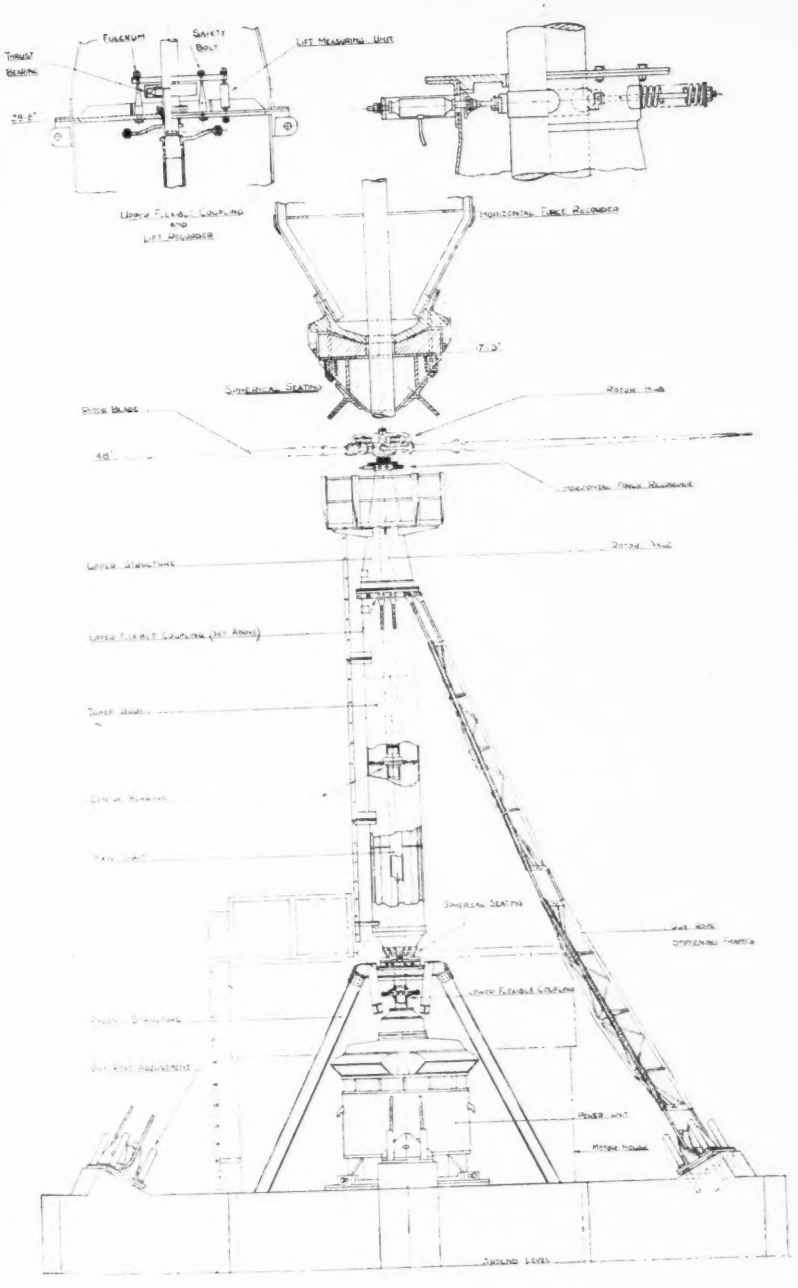



Fig. 1.
General arrangement of rotor tower.

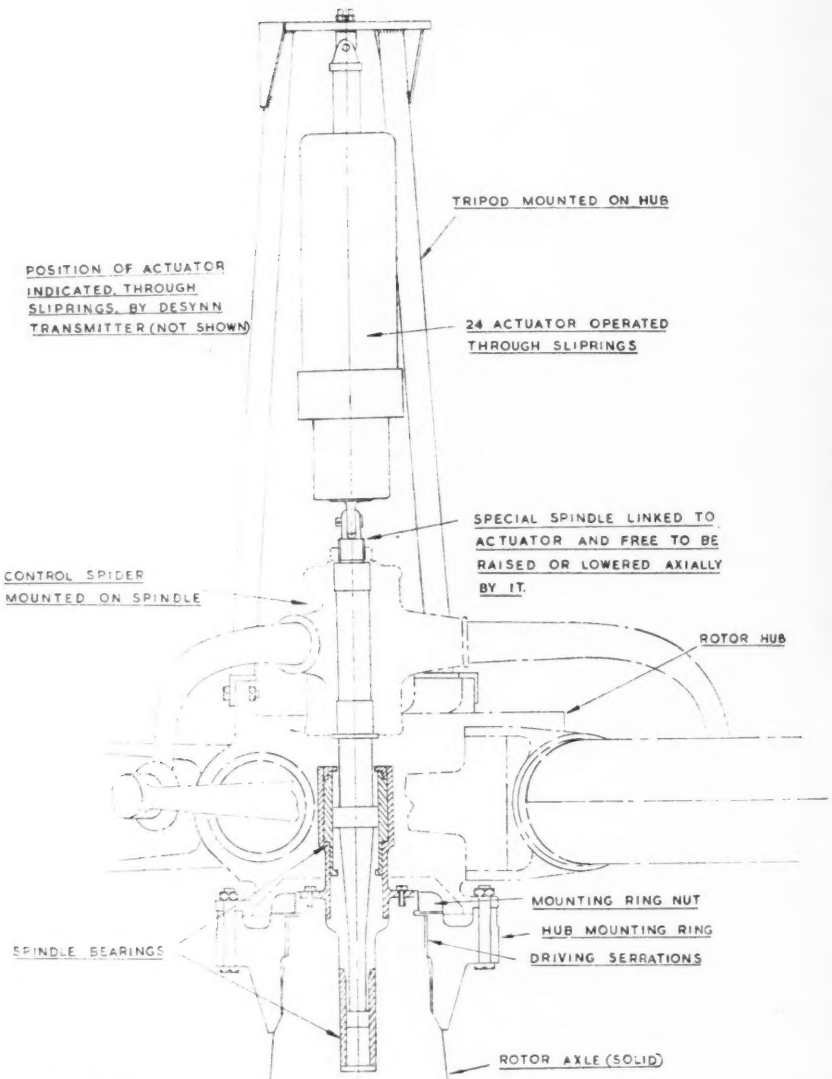

TRIPOD MOUNTED ON HUB
POSITION OF ACTUATOR INDICATED, THROUGH SLIPRINGS, BY DESYNN TRANSMITTER (NOT SHOWN)
24 ACTUATOR OPERATED THROUGH SLIPRINGS
SPECIAL SPINDLE LINKED TO ACTUATOR AND FREE TO BE RAISED OR LOWERED AXIALLY BY IT.
CONTROL SPIDER MOUNTED ON SPINDLE
ROTOR HUB
MOUNTING RING NUT
HUB MOUNTING RING
DRIVING SERRATIONS
SPINDLE BEARINGS
ROTOR AXLE (SOLID)
TOWER STRUCTURE

Fig. 2.
Arrangement for collective pitch variation.

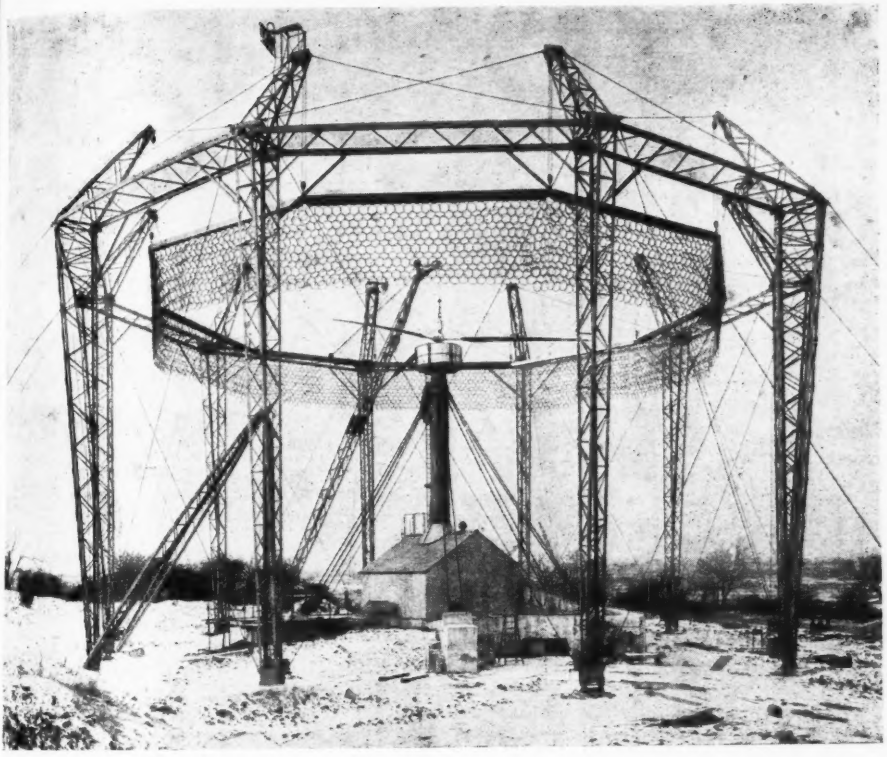


Fig. 3.
View of tower and netting structure.

The axle terminates in a detachable serrated mounting ring, so that the mounting of rotors with differing methods of attachment merely necessitates a change of mounting ring.

*The power unit*

Motive power is supplied by a vertical spindle, direct drive, direct current electric motor (Fig. 5). This motor is rated at 700 h.p. maximum continuous output, which it can develop from its maximum speed of 680 r.p.m. down to 180 r.p.m. It can develop over 1,000 h.p. if required.

Current, taken from the Grid, is transformed to 500 v. and rectified at a works sub-station and conducted to the tower by special cables.

The motor is compound wound. On starting, a large amount of resistance is present in series with the armature, and this is removed in stages until about 170 r.p.m. is reached. Thereafter, power is controlled by varying the strength of the field.

The weight of the armature is carried by a Michell bearing on top of the motor casing. Cooling water is circulated through the oil in the bearing, and also through the liquid series resistance used at low speeds. Forced ventilation of the motor is provided by an independent fan.

A steel-framed building encloses the motor and pylon structure and a manually-operated 40 cwt. crane, with a sixty-foot jib, is installed beside the tower for the mounting and dismounting of rotors.

## 4. INSTRUMENTATION

This section should be regarded as complementary to the two Flight Research

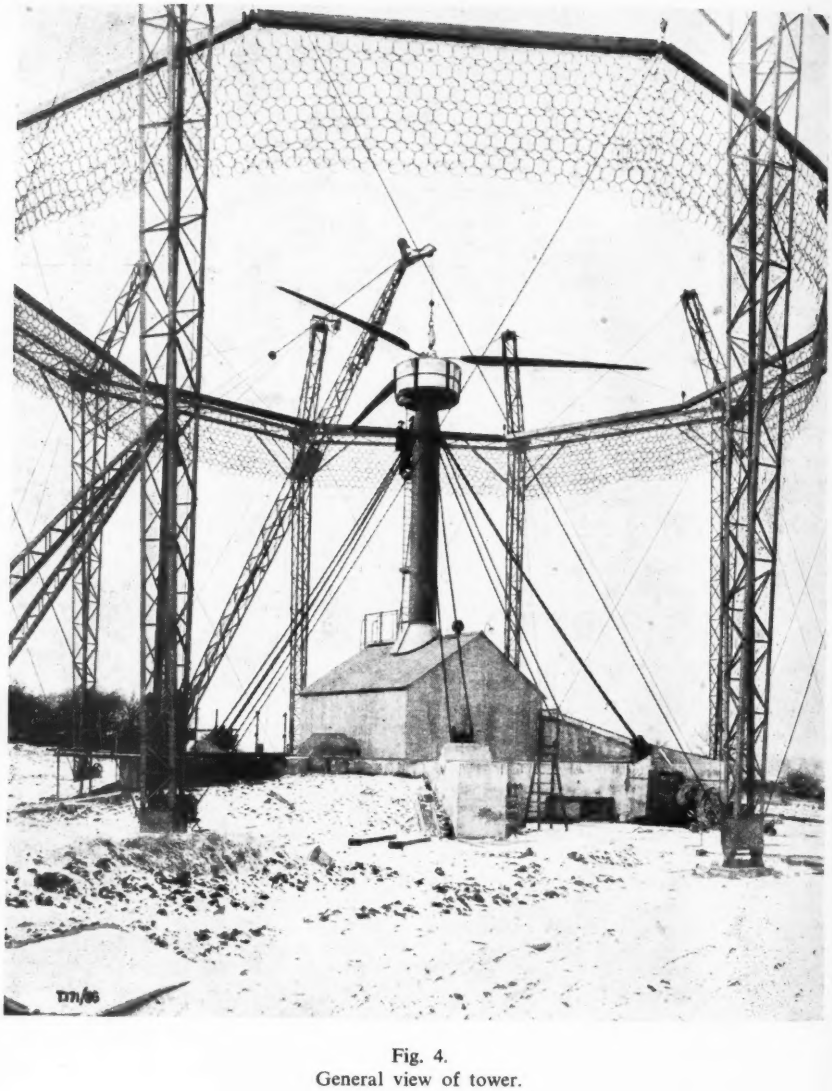


Fig. 4.
General view of tower.

Reports which form the Appendix and which describe the electronic equipment.

*Lift measurement*

The principles employed have already been explained and the arrangement is shown at the top of Fig. 1.

This system has generally been satisfactory but it has been found desirable to increase the sensitivity. Calibration is checked before serious use of the device by applying an axial load at the hub by means of the crane and a spring-balance. This is shown in position on Figs. 1 and 2.

As a further insurance of accuracy, it is proposed to install, in parallel with the

present arrangement, a pressure-sensitive hydraulic capsule system. The capsule will be held in a container so that tension in the container produces compression in the capsule. The container will be mounted between the rotor axle and the tower body in exactly the same way as in the strain-gauge; in fact the lever arm shown in the sketch will be replaced by a symmetrical one, so that the load will be shared equally between the two devices, the present fulcrum becoming a second safety bolt.

*Speed indication*

A small generator driven directly by the main motor and lying underneath it energises a large moving-coil instrument on the control desk (see Fig. 6) which can be read, by interpolation, to one r.p.m.

Accuracy is within one per cent. of the indication and can be checked by the electric counter, which may be seen at the top centre of the desk. The counter is operated once per revolution by a simple slip-ring on the shaft and is used in conjunction with a stop-watch.

*Power measurement*

Power absorbed is found by measuring the electrical power input and referring to the efficiency curves for the motor, plotted by the makers for various speeds and armature voltages.

Armature voltage is measured by a first-grade moving-coil instrument (maximum error 1.5 per cent. of indication) and armature current by a sub-standard instrument (maximum error 0.5 per cent. of indication). Assuming accuracy of the motor calibration, errors in power measurement will not exceed two per cent.

*Out-of-balance forces at the hub*

The out-of-balance force-recording gear is illustrated at the upper right-hand corner of Fig 1. It will be seen that the upper rotor-axle bearing is held by the strain gauge units against two pairs of springs, the two sets lying vertically to one another (see Fig. 7).

Each pair of springs is adjusted to produce a permanent tensile load of 1,000 lb. in the measuring unit so that any out-of-balance force (of less than 1,000 lb.) causes cyclic reductions and increases in this tensile load. These fluctuations are indicated, as described in the Appendix, on two oscilloscopes. From an examination of the film record of the traces, and having regard to the position in azimuth of the blades, the position-indicator ring and pick-up, and the gauges, it is possible to trace an out-of-balance force to any particular blade or blades. So far, such forces have always been attributable to incorrect tracking, which can be dealt with directly.

Calibration of these units is done by varying the tensile force of the springs to known values.

*Measurement of blade pitching moments*

This is treated separately in the Appendix. Figs. 7 and 8 illustrate the principle of measurement; the unstrained arms supporting the coils may be seen clamped to the spider arms. In Fig. 8 two of the units, which are of an early type, have been screened from wind and radiation.

*Position indicator*

The design and operation of the position indicator are discussed in the Appendix.

*Methods of observing blade "tracking"*

Small metal mirrors are attached to the blades near the tips with their faces disposed

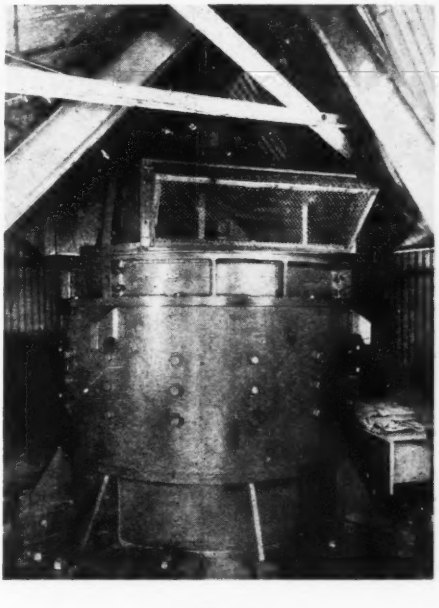


Fig. 5.
700 h.p. motor.

towards the rotor centre so that they are visible to an observer on the tower platform. Two of them have been coloured by transparent lacquers and the third is left plain. The effect of directing the light from a powerful lamp out to the tips during rotation is to produce three variously coloured "streak" reflections in the tip-paths, which can thus be easily identified and their relative heights judged. Appropriate action can then be taken until the tip-paths are coincident.

Solar reflections, when obtainable, have been found to be superior to lamp reflections to such an extent that background is of little importance. This variation of the method has been successfully used in free flight.

The mirrors are slightly cylindrical about a horizontal axis to provide latitude in zenithal "aiming" and to produce a finer streak. This effect is naturally enhanced by a dark background, so a matt black screen has been mounted on the protective netting. Graduations painted on it enable the difference in height of the tips to be judged quantitatively, and also permit general changes in coning angle to be measured.

## 5. PROTECTION PROVIDED AGAINST ROTOR BREAKAGE

The arrangements are shown in Fig. 3. A ring of wire-braced pylons, pin-jointed at the feet, surrounds the tower and supports a circular screen of heavy steel netting. The diameter of the netting is 84 feet, and the whole structure is designed to absorb the greatest amount of energy likely to be stored in a rotor blade which has left the hub at a high rotational speed.

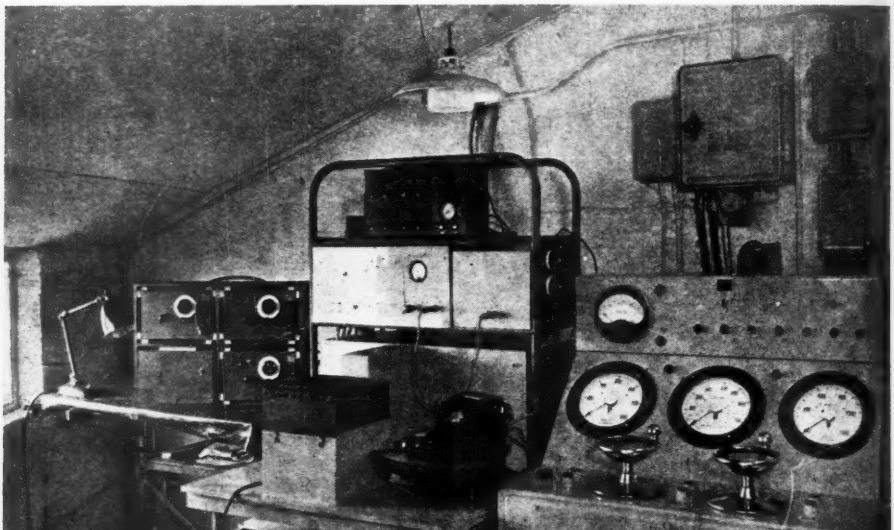



Fig. 6.
Control and recording equipment.
*From left to right*: Frequency modulation units (old type) for blade moment measurement; five channel oscilloscope unit, with strain gauge bridges on top and recording camera in front; motor control panel.

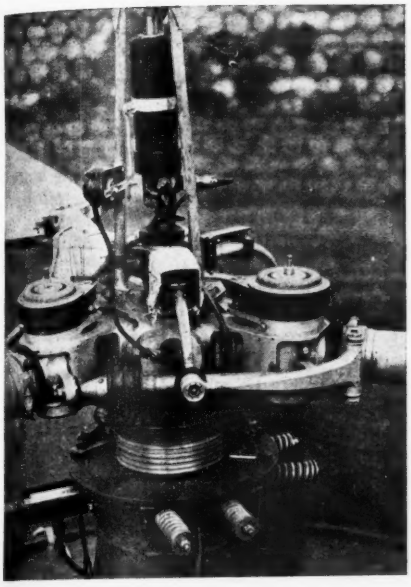


Fig. 7.
General view of hub showing, from top to bottom, collective pitch changing actuator and Desynn transmitter, blade moment gauges on spider arms, rotor hub, slip-rings and brushes, and out-of-balance gauges and springs.

The height of the netting at each pylon may be adjusted by means of a hand-winch at the foot, making it a simple matter to move the screen into the probable path of blades and other pieces thrown from any type of configuration. The roof of the control-room is protected from heavy fragments which might strike the netting and fall back, by a layer of steel fabric stretched twelve inches above it.

As the electric motor is compound wound, there is no danger of an uncontrolled speed increase in the event of sudden removal of the load.

For improved observation of flutter and general rotor behaviour a post has been established on higher ground nearby, which is in the plane of the rotor disc. A telephone and duplicate contact-breaker button are provided here.

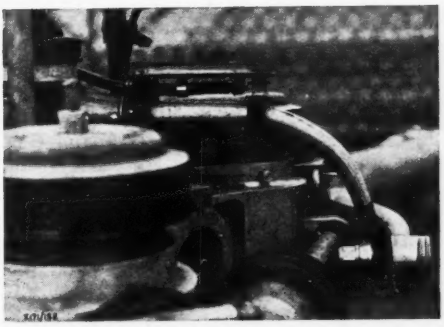
## 6. TESTS

The tower has been used for calibration of a brake fan, rotor endurance testing, flutter research and proof testing, over-speed testing, and the development of tracking methods. It has provided valuable data on all these points except the last, for which the measuring equipment originally provided was found to be unsatisfactory. It is believed that the modified equipment described in the Appendix and now being installed, should enable this function also to be carried out successfully.

## 7. CONCLUSIONS REACHED AND FURTHER DEVELOPMENTS CONTEMPLATED

*Motor and drive*

In general, the electric motor is satisfactory. The speed, once set, will remain constant indefinitely, affected only by the normal slight variations in supply voltage,



Method of measuring bending of spider arm with inductance gap gauge. (See appendix).

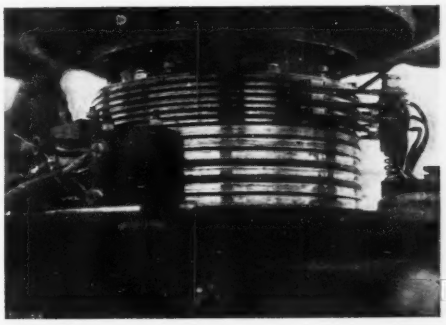


Fig. 8.
Slip-rings.
Upper rings supply actuator and carry signal from Desynn indicator. Lower rings are for frequency-modulated circuits.

which can be corrected. In the event of a sudden removal of load, such as rotor failure, there is no tendency towards increase in speed. The quietness of the motor is another good point.

The armature, as mentioned earlier, is suspended from a Michell bearing. This type of bearing, if loaded when not rotating, has a rather high static frictional torque, and the voltage which has to be applied to overcome this produces a higher starting acceleration than might be desirable with some rotors. There are several ways in which this might be avoided, but the one most favoured is the initial breaking down of the static friction by a small servo-motor fitted with a free-wheel, as this could also be used to turn the rotor through small displacements for adjustment and other purposes. Design of this refinement is proceeding.

Another piece of equipment which it is hoped to fit shortly is a friction brake. The high inertia of rotor and armature, results in a long interval between the pressing of the stop-button and the cessation of motion, which can be embarrassing when testing involves a large number of starts and stops. It will also have a certain emergency value.

*Hub and pitch-change mechanism*

The present system of collective pitch variation is in itself satisfactory, but it is also highly desirable that provision for cyclic pitch variation should be made. Cyclic pitch change in the conventional sense, *i.e.* pitch change with a frequency of one cycle/rev. and with the three blades phased at 120° to one another, serves only to tilt the disc and thus provide motion of the flapping pins, but does not cause movement at the drag pins or fluctuating stresses in the blades. Proposals are therefore being considered for making rapid changes in the *collective* pitch of the blades at frequencies of the order of twice rotor speed, thus causing both flapping and dragging motion and producing fluctuating blade stresses.

*Future test work*

While details of the future programme will necessarily depend on the time at which the various improved or additional items are obtained and also on the availability of rotors, the following additional types of testing are proposed.

Strain gauging of blades and measurement of blade movements. The tower will be used to help in developing the technique for such measurements, as well as for such quantitative testing as can be done at low forward speeds (wind speed).

Endurance testing of hubs and blades, including fatigue tests.

Further flutter research.

Aerodynamic research, including investigation of mean induced velocity and its cyclic variation.

## APPENDIX

## THE MEASUREMENT OF BLADE PITCHING MOMENTS ON THE HELICOPTER TOWER

*by* H. HILL

(*The Bristol Aeroplane Co. Ltd.*)

### DESCRIPTION OF EQUIPMENT

It was desired to measure the pitching moments of individual blades on the helicopter tower to an accuracy of ± 10 lb. in. In view of the low stresses involved and the fact that slip rings would have to be used, it was decided to use variable inductance gauges with frequency modulation amplifiers. Each gauge was mounted on one end of an unstrained arm which paralleled the spider through which all blade root moments are resisted. As long a length of spider arm as possible was used so that the gap variation produced between the unstrained arm and the spider arm would be as large as possible when bending moments are applied to the latter. The gap variation was used to produce an inductance change in a coil which was wound on an E-shaped iron dust core and mounted on the unstrained arm. This core had another similar core adjacent to it which was fixed to the spider arm. These coils were coupled to their oscillators through slip rings and co-axial cables. Each oscillator output was fed through an amplifier and a limiter stage to a discriminator and thence to a push-pull output stage. The output of each was fed on to a cathode ray tube and the deflection of the spot measured. The cathode ray tubes were employed so that any tendency to flutter could be observed.

### PERFORMANCE

It was found that the sensivity of the units was entirely adequate but that the stability was poor. The drift imposed a frequent and laborious zero checking procedure. This drift was thought to be caused by temperature effects on the metal of the measuring arms, so new arms were made and fitted which reduced the magnitude of the drift somewhat, but it was still too large. Attention was turned to the slip rings and experiments with different materials for brushes and rings gave promise. Unfortunately, the best materials for reducing drift were not the materials most suited to resist wear and atmospheric corrosion. It was decided to make a change by moving the location of the oscillator in relation to the slip rings. This would mean that slip ring variables would cease to affect the units. Experiments were conducted with miniature oscillator units mounted on the unstrained arms and fed with H.T. and L.T. current through the slip rings and a large number of types of oscillator, each with many types of valve, were tried. The requirement to be fulfilled was high frequency stability under the following conditions:—

(*a*) Applied acceleration forces up to 10g.

(*b*) Air velocities up to 20 ft./sec.

(*c*) Large variations of H.T. and L.T. voltage; it must also have small physical dimensions.

A design of oscillator which fulfills these requirements has now been evolved and a complete set of equipment has been constructed which is outlined below.

### LATEST EQUIPMENT

A single valve oscillator is mounted on the measuring arm and is fed with H.T. and L.T. current through slip rings. The output of each oscillator is fed through a slip ring to an amplifier, limiter and discriminator stage. The amplified output from the discriminator stage is indicated on a pointer indicator which will be calibrated directly in pound inches. The three amplifier units each feed through a coupling condenser on to a single cathode ray tube, which will thus show any flutter. A four-position switch enables the cathode ray tube to be fed by any amplifier singly, or by all three simultaneously.

The long term and short term stability are both high, the drift being not more than 10 lb. in. in two hours.

## ELECTRONIC INSTRUMENTATION

*by* A. E. CAIN

(*The Bristol Aeroplane Co. Ltd.*)

The electronic equipment installed in the tower was designed to measure:—

(*i*) the total lift of the rotor,

(*ii*) two horizontal forces at the hub at right angles to one another,

(*iii*) the angular position of the rotor, and

(*iv*) the individual pitching moments of the three rotor blades.

The equipment to measure the total lift of the rotor and the two horizontal forces consists of:—

(1) *A 5 tube oscillograph* using $1\frac{1}{2}$ in. cathode ray tubes. The tubes are arranged in a vertical line and are photographed on 70 mm. paper film, a camera and camera hood being provided as part of the oscilloscope. The angular position of the rotor is indicated on the paper by means of an N.S.P.I. stroboscopic lamp which is mounted in a tube which has a vertical slot .01 in. wide and $\frac{1}{4}$ in. long cut in its side opposite the light column. The slot is focused on to the paper by means of a mirror mounted between No. 2 and No. 3 cathode ray tubes. Thus, each time the lamp is triggered and discharges its associated condenser, a vertical line is obtained on the paper.

The voltage impulse necessary to fire the lamp is obtained by means of a disc mounted on the motor shaft. The disc has slots $\frac{1}{16}$ in. wide by $\frac{1}{16}$ in. deep cut in its periphery. A magnet and coil are mounted close to the disc so that each time a slot passes the end of the magnet a voltage is induced in the coil; this voltage is amplified by a pentode to the amplitude required to trigger the N.S.P.I.

Seven slots are cut in the disc, six of which are spaced at 60° intervals and the seventh intermediate between two of the others. Thus in one revolution the markings on the film space out two 30° intervals followed by five 60° intervals. By this means the angular position of the rotor is known.

Time marking at 50 c.p.s. is also provided by an N.S.P.I. lamp mounted in an identical manner to the position indicating lamp. In

this instance the integrated output of a multi-vibrator is used to trigger the N.S.P.I.

Each tube has its own amplifier and the amplifiers are resistance-capacity coupled, with voltage feed-back. The valves used in each amplifier are two 6SJ7's followed by a 6C5 in the output stage and each amplifier has a maximum gain of 10,000 and is fitted with an attenuator, to also give gains of 5,000, 2,500, 1,250, 625 and 312.5. The H.T. supply to the amplifiers and the multi-vibrator is stabilised. A switch is provided to deflect the beams from the C.R.O. tubes when no recording is taking place and this keeps screen burning to a minimum.

The cathode ray tubes, amplifiers, and strobotron lamps, together with their power supplies, are built into one case.

(2) *An oscillator and power amplifier* to energise the strain gauges.

The oscillator is of the resistance capacity type and is from a design by Dr. F. Aughtie of the N.P.L. Three valves are used in the oscillator and as each valve and its associated network gives 120° phase shift, it is essentially a three-phase oscillator. This oscillator was used because of its good wave form at the low frequency required.

The voltage output from the oscillator is fed to the power amplifier and there converted to sufficient power to energise the strain gauges. Three separate outputs are provided from the amplifier, enabling each strain gauge circuit to be independent of the other two.

(3) *Strain gauge bridges and strain gauges.*

The strain gauge bridge consists of two strain gauges, two fixed resistance arms and a variable resistor of one ohm for balancing. Two adjustable iron slugs which slide inside coils connected in the two fixed arms are used for phase correction. One strain gauge is the strain measuring arm, while the other is a temperature compensating gauge. These two gauges are connected to the remainder of the bridge by means of 80 ft. lengths of trimet cable. The gauge which measures the strain is cemented on to a tensile test piece and this gauge causes an out-of-balance potential in its own bridge proportional to the strain in the tensile test piece. This out-of-balance potential is indicated on the cathode ray tube after being amplified. Then, since the strain is proportional to the force being applied to the tensile piece, we have a measure of the forces being applied.

(4) *A mains control unit* consisting of a "Variac transformer" and a voltage indicating meter is installed so that a known and constant mains voltage can be fed to the equipment.

(5) *The pitching moments of the three rotor blades.*

A description of this equipment has already been given. Modifications completed recently include the following.

The slip rings assembly has been modified so that when blade pitching moments are not being measured the 12 slip rings can be used for 10 channel strain gauging of the blades. As only 5 C.R.O. tubes are available, a three-position selector unit has been designed for switching purposes. The first position connects the three tensile gauges to the amplifiers of the three top tubes when blade balancing is required; the second position connects five of the strain gauges to the C.R.O., and the third position another five gauges. By this means 10 strain gauges' positions can be recorded without changing rotor conditions. As the gauges to be used for strain gauging the blades will be of the order of 2,400 ohms, the strain gauges on the tensile test pieces have been changed from 200 ohms to 2,400 ohms. This enables the existing oscillator and amplifier to be used for the double purpose. The output of the energising amplifier has been increased sufficiently to energise the higher resistance gauges.

The manufacture has begun of 10 additional strain gauge bridges to be used on blade gauging. These bridges will be complete with phase correction network.

# THE GENERAL THEORY OF CYLINDRICAL AND CONICAL TUBES UNDER TORSION AND BENDING LOADS

## Single and Many Cell Tubes of Arbitrary Cross-Section with Rigid Diaphrams

## PART VI

by

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## 6. OPEN TUBES, SERIES OF JOINED TUBES, AND NUMERICAL EXAMPLE

SUMMARY

Parts 1 to 5 of this paper (February, September and November 1947 issues of the JOURNAL) investigated the stresses and deformations of closed tubes in which the thicknesses were governed by the $t_s$* and $t$* laws. In the present part, the analysis is extended to multi-cell tubes with openings, open tubes with or without St. Venant torsional stiffness, and to tubes formed by joining elements of different cross-sections. To illustrate the theory a numerical example of the stressing of a four-boom wing consisting of seven joined elements is fully worked out. Finally, an appendix gives practical methods of dealing with tubes which do not conform to the $t_s$* and $t$* laws, and of finding approximate solutions for four-boom tubes with direct stress carrying covers.

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## 6.1. Introduction

The general analysis of multi-cell tubes with openings presents no intrinsically new problems and only the determination of the eigenvalues $\lambda_i$ and eigenfunctions $h_i$ requires some discussion. This is given in Section 6.4 where also the particular case of an $n$-boom multi-cell tube of the type of Fig. 16 with an opening is analysed. It is shown how the equivalent tube method of Section 5.7.4 is, by a limiting process, still applicable in the present case. For a four-boom multi-cell tube the eigenvalue and corresponding eigenfunction are given in explicit form.

Section 6.5 begins with an investigation of the open tube with St. Venant torsion constant $J$. The analysis is made by representing the St. Venant torsional stiffness by a closed tube effective only in shear, and thus the problem is basically the same as that of Section 6.4. For a multi-boom tube with $J$ the eigenvalues and eigenfunctions are completely determined. It is shown also that the multi-cell tube of Section 6.4 may be made to depend on an equivalent open tube with $J$, and in this way the detailed results for the multi-cell tube are best obtained. In wing structures the St. Venant torsional stiffness of open sections is generally small enough to neglect. Thus the open tube with $J=0$ is of particular practical importance. The complete theory for such tubes is developed from the case with $J$ not zero by a limiting process, which shows that the first eigenload degenerates into the Wagner[23] - Kappus[24] torsion-bending stress system for $J=0$ as generalised for conical tubes. This stress system is statically equivalent to the applied torque about the shear centre, and the total stresses take the form,

$$\left.\begin{aligned}\sigma &= \sigma_E + \sigma_\Gamma + \sum_i T_{Bio}\Theta_i G_{\sigma i} L_{\sigma i} \\ \tau &= \tau_E + \tau_\Gamma + \sum_i T_{Bio}\Theta_i G_{\tau i} L_{\tau i}\end{aligned}\right\} \quad . \quad . \quad . \quad (627)$$

where $\sigma_\Gamma$ and $\tau_\Gamma$ are the torsion-bending stresses. The formulæ for $\sigma_\Gamma$ and $\tau_\Gamma$ bear a striking analogy to those for $\sigma_E$ and $\tau_E$. The torsion-bending stresses and the $(n-4)$ eigenvalues and eigenfunctions are developed for $n$-boom open tubes of arbitrary and symmetrical cross-section. The four-boom open tube has no eigen-

functions and the torsion-bending stresses take a particularly simple form. Section 6.5 concludes with an investigation of the deformations of an open tube with $J=0$, and it is shown that the shear deformations, which are neglected by the Wagner-Kappus theory, are important in short *open* tubes. Thus a four-boom open cantilever will twist even when loaded through the shear centre (see Proceedings of Aeronautical Conference, 1947[27]).

Section 6.6 deals with the problem of wing structures formed by joining several tube elements of different cross-sections but in each of which the $t^*$, $t_s{}^*$ laws are satisfied. The general boundary conditions for multi-boom tubes are developed in Sections 6.6.1 and 6.6.2. In Section 6.6.3 the boundary conditions for four-boom closed or open elements are shown to lead to "three moment" equations in the unknown self-equilibrating load systems at the ends of each element.

In Section 6.7 explicit formulæ for the engineers' theory shear stress distribution are given for closed or open multi-cell tubes of the type of Fig. 16. Also presented is a general method for determining the positions of the centre of the twist $S_i$ relative to the shear centre $E_s$.

The final section 6.8 is a numerical example of the stressing of a tube built up by joining seven four-boom elements. The overall dimensions and the loading are based on an actual wing under a symmetrical gust but some changes in the structure have been made to make the example more illustrative of all types of four-boom cross-sections. All the computations are arranged in tabular form.

The appendix, which concludes the whole paper, discusses practical methods of applying the theory to tubes in which the similarity laws of the thicknesses are not followed, *i.e.* the $t^*$ and $t_s{}^*$ variations are not true. The estimation of the axial constraint stresses in such cases is a much more subtle problem than is generally realised, since they cannot be calculated by taking the torque about an axis solely defined by the tube geometry. Also given are approximate methods for obtaining the stress distribution in four-boom tubes with direct stress-carrying covers.

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## 6.2. ADDITIONAL NOTATION

$D_x = \int_0^s t_s \bar{x}\, ds$

$D_y = \int_0^s t_s \bar{y}\, ds$

$J$ — St. Venant torsion constant at root cross-section of open tube.

$\Gamma = 4\int_C t_s (A_s)^2\, ds$ — Torsion-bending constant at root cross-section of open tube.

$F_\Gamma = g_\Gamma h_\Gamma$ — Stress function for torsion-bending stress distribution in open tubes.

$g_\Gamma$ — Function of $r$.

$h_\Gamma$ — Function of $s$.

$\sigma_\Gamma$, $\tau_\Gamma$ — Direct and shear stresses of the torsion-bending stress distribution.

$D_\Gamma$ — Constant of integration.

$W_\Gamma{}^*$ — Warping function due to torsion-bending stress distribution in open tubes (defined by equation (631)).

$\Theta_i = \frac{1}{2A}\oint_s \frac{h_i}{t}\, ds$

$b_{xi}=\xi_i\Theta_i$
$b_{yi}=\eta_i\Theta_i$
$X_{qj}$, $Y_{qj}$ — Defined by equations (636).
$A_{qj}$ — Defined by equation (642).
$c_{iqj}$ — Defined by equation (637).
$\int_c$ — Integral over wall of open tube.
Suffix $q$ — Refers to values associated with the $q$th element in a series of joined tube elements.
Suffix $J$ — Refers to values of closed tube simulating the St. Venant torsional stiffness of open section.

*The following notation applies only to four-boom tubes.*

$$\Xi_{41}=\frac{1}{2}\,|\,12,\ 23,\ 34\,|$$

$\Psi=\sum_1^4 \delta_{12}\,(\bar{A}_{34})^2$ — Applies to single cell or multi-cell tube without openings.

$\Psi=\sum_1^3 \delta_{12}\,(\bar{A}_{34})^2+(\Xi_{41}/A^m)^2$ — Applies to multi-cell tube with opening (41).

$\Lambda=2A^*/\mu\Psi^{1/2}$
$\Theta_D=\Theta\Psi^{1/2}$
$P_o$ — Boom load function defining the self equilibrating end load system applied at the root of a closed tube with tip free.
$P_c$ — Boom load function defining the self-equilibrating end load system applied at the tip of a closed tube with root free,
or
Boom load function defining the self-equilibrating end load system applied at the tip of an open tube with root built-in.
$P_T$ — Boom load function due to axial constraint stresses in a closed tube with root built-in and tip free.
$P_\Gamma$ — Boom load function due to torsion-bending stresses in an open tube with root built-in and tip free.
$w$ — Warping function of a closed tube defined by equation (658).
$w_\Gamma$ — Warping function of an open tube defined by equation (658*a*).
$\overset{o}{w}$ — Warping function due to $P_o=1$ (defined by equation (664)).
$\overset{c}{w}$ — Warping function due to $P_c=1$ (defined by equation (664)).

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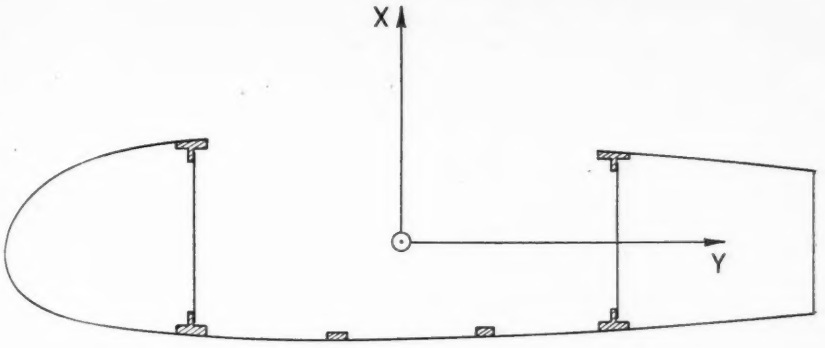


Fig. 30.
Multi-cell tube with opening.

### 6.4. THE MULTI-CELL TUBE WITH AT LEAST ONE OPEN CELL AND AT LEAST ONE CLOSED CELL

Multi-cell tubes with certain cells open but in which there is at least one closed cell (see Fig. 30) present no new difficulties. It is only necessary to discuss the determination of the $h$-functions after which the analysis is precisely as in Parts I to V.

#### 6.4.1. *The Bredt-Batho function* $h_B$

Consider first the simple example of a nose cell with an open rear section as shown in Fig. 31. It is obvious that $h_B$ exists only in the closed nose cell $A_I$ and hence,

$$\left.\begin{array}{l} h_B = 1 \text{ in walls 16 and 61} \\ h_B = 0 \text{ in all other walls} \end{array}\right\} \quad . \quad . \quad . \quad . \quad . \quad (527)$$

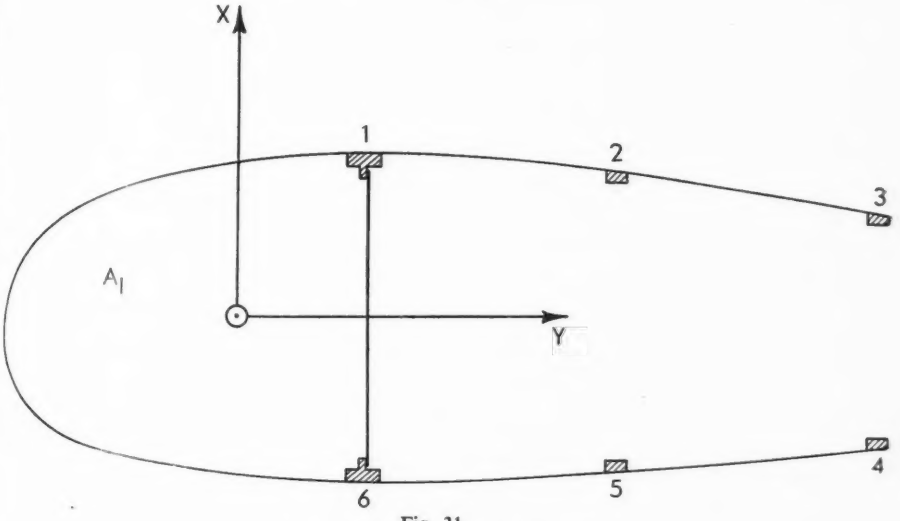


Fig. 31.
Tube with one closed and one open cell.

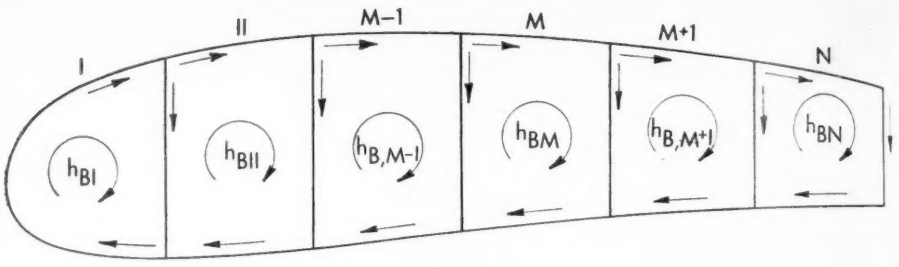


Fig. 32.
Determination of $h_B$-distribution in a multi-cell tube.

Note that formula (268) then reads in the present case,

$$\frac{dg_B}{dr} = -\frac{T_{BE}}{2A_1\rho^2} \quad . \quad . \quad . \quad . \quad . \quad (527a)$$

The multi-cell tube with openings is best discussed after considering the completely closed multi-cell tube of Fig. 32. The solution to this problem has been indicated in Section 4.1.2, but it is preferable to use the method adopted for the calculation of the $h_X$ distribution in Section 5.4.3. The $h_B$ distribution for an $N$-cell closed tube may be represented by the $N$ distributions

$$h_{BI}, h_{BII}, \ldots\ldots, h_{BM}, \ldots\ldots, h_{BN}$$

about each cell as shown in Fig 32.

Thus the $h_B$-distribution in the walls of the $M$th-cell will be as follows:
in the outer walls,

$$h_B = h_{BM} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (528a)$$

in the left hand wall,

$$h_B = h_{B(M-1)} - h_{BM} \quad . \quad . \quad . \quad . \quad . \quad (528b)$$

and in the right hand wall,

$$h_B = h_{BM} - h_{B(M+1)} \quad . \quad . \quad . \quad . \quad . \quad (528c)$$

The rate of twist of the $M$th cell is proportional to,

$$C_B = \frac{1}{A_M} \oint_M \frac{h_B}{t}\, ds \quad . \quad . \quad . \quad . \quad . \quad (529)$$

where,

$$\oint_M \frac{h_B}{t} ds = -h_{B(M-1)} \int_{M-1,\,M} \frac{ds}{t} + h_{BM} \oint_M \frac{ds}{t} - h_{B(M+1)} \int_{M,\,M+1} \frac{ds}{t} \quad . \quad . \quad (529a)$$

The integrals on the right hand side of (529$a$) are identical to the corresponding terms in equation (289$a$). Since $C_B$ is the same for all cells one obtains from equations (529) and (529$a$) $N$ equations:—

$$\left.\begin{aligned} h_{BI} \oint_I \frac{ds}{t} - h_{BII} \int_{I,\,II} \frac{ds}{t} &= A_1 C_B \\ \text{- - - - - - - - - - - - - - - - - -} & \\ -h_{B(M-1)} \int_{M-1,\,M} \frac{ds}{t} + h_{BM} \oint_M \frac{ds}{t} - h_{B(M+1)} \int_{M,\,M+1} \frac{ds}{t} &= A_M C_B \\ \text{- - - - - - - - - - - - - - - - - -} & \\ -h_{B(N-1)} \int_{N-1,\,N} \frac{ds}{t} + h_{BN} \oint_N \frac{ds}{t} &= A_N C_B \end{aligned}\right\} \quad . \quad . \quad (530)$$

The solution of equation (530) may be written in the form,

$$h_{BM}=C_B\sum_{R=1}^{N}a_{MR}A_R \quad . \quad . \quad . \quad . \quad . \quad (531)$$

where the coefficients $a_{MR}$ are the same as those occurring in equation (291). The unknown $C_B$ is determined from the equilibrium condition (99) which may be written,

$$\sum_{R=1}^{N}h_{BR}A_R=\sum_{R=I}^{N}A_R=A \quad . \quad . \quad . \quad . \quad . \quad (532)$$

If the tube has open cells the above analysis applies with little modification. Thus if the cells

$$P,\ Q,\ R,\ — — — —$$

are open, one omits the

$$P\text{th},\ Q\text{th},\ R\text{th},\ — — — —$$

equations in (530) and in equations (531) and (532) the terms involving,

$$h_{BP},\ h_{BQ},\ h_{BR},\ — — — —$$
$$A_P,\ A_Q,\ A_R,\ — — — —$$

do not exist. Note that the area $A$ now becomes the total area of all the *closed* cells.

6.4.2. *The $h_X$ and $h_Y$-functions*

The determination of the $h_X$ and $h_Y$-functions for multi-cell tubes with openings follows from the analysis given in Section 5.4.3 for a completely closed tube, applying the same modifications as have just been given in Section 6.4.1 for the $h_B$ distribution. For the length $S$ in equations (275) and (275*a*) any suitable linear dimension may be used. The shear centre is given by equations (293*a*) and (293*b*), only the closed cells appearing in the summations,

$$2\Sigma A_M h_{XM} \text{ and } 2\Sigma A_M h_{YM}$$

Formula (294) for the rate of twist applies again in the present case with the understanding that $A$ is the total area of the closed cells, and that

$$\oint_S \frac{h_B}{t}ds$$

is taken over all external walls of the closed cells.

6.4.3. *The $h_i$-functions in general*

The calculation of the eigenfunctions for a multi-cell tube with openings does not present any essentially new problems. The equilbrium conditions (54) are still valid and the boundary conditions at the joints of the cell walls are of the type (63). The conditions at such an isolated boom as 4 in Fig. 31 may be written,

$$h_{4+}=B_4\left(\frac{1}{t_s}\frac{dh}{ds}\right)_{4+} \quad . \quad . \quad . \quad . \quad . \quad (533)$$

or if there is no boom at 4,

$$h_{4+}=0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (533a)$$

For a multi-boom tube with shear-carrying walls equations of the type (66*a*) and (72) are again directly applicable.

In the expression

$$\frac{1}{2A}\oint_S \frac{h_i}{t}ds$$

the same remarks apply to the area $A$ and the path of integration as made at the end of Section 6.4.2. It will be convenient in what follows to introduce the notation

$$\Theta_i=\frac{1}{2A}\oint_S \frac{h_i}{t}ds=\frac{1}{2A_M}\oint_M \frac{h_i}{t}ds \quad . \quad . \quad . \quad . \quad (534)$$

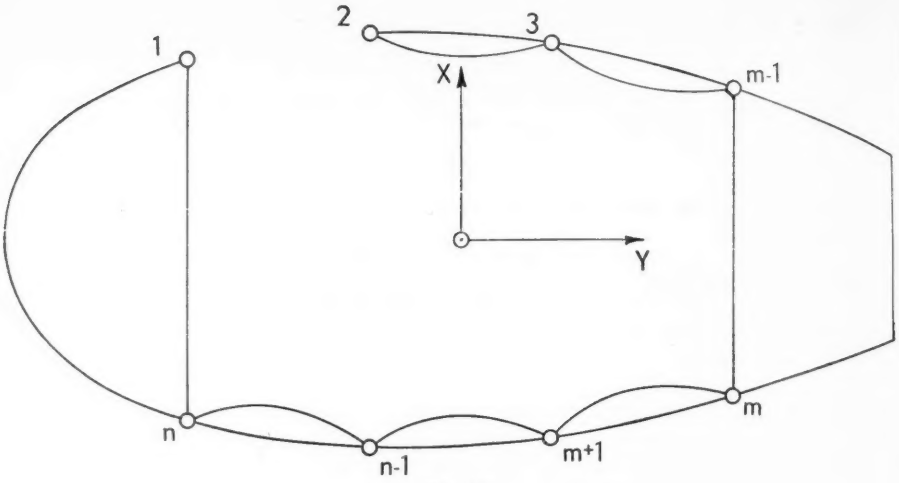


Fig. 33.
Multi-cell $n$-boom tube with opening.

The function $\Theta_i$ determines the influence of the cross-sectional dimensions of the root on the rate of twist of the tube under the $i$th self-equilibrating end load system. With this notation the differential equation (45) may be written,

$$\frac{d}{ds}\left(\frac{1}{t_s}\frac{dh_i}{ds}\right)+\lambda_i^2\frac{h_i}{t}=\lambda_i^2\,(p-\xi_i\cos\psi+\eta_i\sin\psi)\,\Theta_i=\lambda_i^2\,\hat{p}_i\Theta_i \quad . \quad (535)$$

The notation $\Theta_i$ is used throughout Part 6 and many of the previous results are better expressed in terms of this notation. For example the stress equations (314) are preferably written,

$$\left.\begin{aligned}\sigma&=\sigma_E+\sum_i T_{Bi}\Theta_i G_{\sigma i}L_{\sigma i}\\ \tau&=\tau_B+\tau_E+\sum_i T_{Bi}\Theta_i G_{\tau i}L_{\tau i}\end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad (314a)$$

With this notation the cross-distribution of the stresses is always expressed in terms of the $L$-functions instead of the $H$-functions, and the possible error of associating an incorrect $A$ with the $\oint_S$ is avoided.

### 6.4.4. *The $h_i$-functions for an n-boom multi-cell tube with an opening.*

The case now to be considered in detail is the multi-cell tube of the type shown in Fig. 33, *i.e.* the tube of Fig. 16 with an opening. Such a tube may be regarded as a multi-cell tube in which the thickness of the wall between booms 1 and 2 tends to zero. Hence it may be treated by the method of Section 4.3.2 as generalised in Section 5.7.4 with

$$\lim \delta^o_{12}=\lim \delta_{12}=\infty \quad . \quad . \quad . \quad . \quad . \quad (536)$$

and $A^o_{12}$ any finite value.

The reduction process of Section 5.7.4. will then be modified as follows:—

(*a*) The length thickness ratios $\delta_{r,\,r+1}$ of all other walls of the equivalent tube will still be found from (206*a*).

(*b*) The areas $A^e_{r,\,r+1}$ are found from (207*a*) noting that $A^e_{12}$, which is not required, reduces to the arbitrary $A^o_{12}$. Thus $A^e$ is arbitrary but finite.

The area $A^m$ is obtained from the formula (210*a*)

$$(A^m)^2=\sum_2^n \frac{(A^{oi}{}_{r,\,r+1})^2}{\delta^o{}_{r,\,r+1}+\delta^i{}_{r,\,r+1}} \quad . \quad . \quad . \quad . \quad (537)$$

(*c*) The value of $K$ given in (214*a*) reduces to,

$$K=-A^m \lim (\delta_{12})^{-1/2}=0 \quad . \quad . \quad . \quad . \quad (538)$$

(*d*) The swept areas $A_{r,\,r+1}$ of the walls of the equivalent tube become,

$$A_{12}=-\lim K\delta_{12}=A^m \lim (\delta_{12})^{1/2}=\infty \quad . \quad . \quad . \quad (539)$$

and for $r \neq 1$,

$$A_{r,\,r+1}=A^e{}_{r,\,r+1} \quad . \quad . \quad . \quad . \quad . \quad (539a)$$

Note also that the area $A$ of the equivalent tube is,

$$A=-\lim K\sum_1^n \delta_{r,\,r+1}=A_{12}=\infty \quad . \quad . \quad . \quad . \quad (539b)$$

Thus the equivalent tube has an infinite area with wall (12) of infinite length but of infinitesimal thickness, such that the torsional stiffness of the equivalent tube remains finite.

(*e*) The values of $\lambda_i{}^2$, $h_i$, $\Theta_i$, $\xi_i$ and $\eta_i$ of the equivalent tube may be obtained by a limiting process from the analysis of Sections 4.3.2 and 5.6.4 for the four-boom tube, and Sections 5.7.1-3 for the $n$-boom tube. The details of this analysis for the four-boom tube will be given in Section 6.4.5, but for the $n$-boom tube it is more elegant to derive the results from the analysis for an open tube with St. Venant torsion constant $J$. This analysis will appear in Section 6.5.4 and the equivalent value of $J$ will be found below.

Note that from equation (205*a*)

$$\lim h_{12}=2\Theta_i \lim (A_{12}/\delta_{12})=2\Theta_i A^m \lim (\delta_{12})^{-1/2}=0 \quad . \quad . \quad (540)$$

(*f*) From (211*a*)

$$h'_{r,\,r+1}=h_{r,\,r+1} \quad . \quad . \quad . \quad . \quad . \quad (541)$$

and finally for $r \neq 1$,

$$\left.\begin{aligned} h^o{}_{r,\,r+1} &= \frac{\delta^i{}_{r,\,r+1}}{\delta^o{}_{r,\,r+1}+\delta^i{}_{r,\,r+1}} h_{r,\,r+1}+\frac{2A^{oi}{}_{r,\,r+1}}{\delta^o{}_{r,\,r+1}+\delta^i{}_{r,\,r+1}}\Theta \\ h^i{}_{r,\,r+1} &= \frac{\delta^o{}_{r,\,r+1}}{\delta^o{}_{r,\,r+1}+\delta^i{}_{r,\,r+1}} h_{r,\,r+1}-\frac{2A^{oi}{}_{r,\,r+1}}{\delta^o{}_{r,\,r+1}+\delta^i{}_{r,\,r+1}}\Theta \end{aligned}\right\} \quad . \quad (542)$$

To show that the $n$-boom equivalent tube of the above analysis is physically identical to an $n$-boom open tube with $J$ one notes first that the term $h_{12}$ may be omitted from the equations of the type (66*a*) and the two shear equilibrium equations (67). The zero torque condition becomes,

$$\sum_2^n h_{r,\,r+1}A_{r,\,r+1}+\lim (h_{12}A_{12})=0$$

or with (539) and (540)

$$\sum_2^n h_{r,\,r+1}A_{r,\,r+1}+2(A^m)^2\Theta=0 \quad . \quad . \quad . \quad . \quad (543)$$

The normalisation condition (70) may be written,

$$\sum_2^n \delta_{r,\,r+1}(h_{r,\,r+1})^2+\lim \delta_{12}(h_{12})^2=1$$

or with (540)

$$\sum_2^n \delta_{r,\,r+1}(h_{r,\,r+1})^2+4(A^m)^2\Theta^2=1 \quad . \quad . \quad . \quad . \quad (544)$$

Comparing equations (543) and (544) with (559) and (560) of 6.5.3 it follows that the equivalent value of $J$ is,

$$J=4(A^m)^2 \quad . \quad . \quad . \quad . \quad . \quad . \quad (545)$$

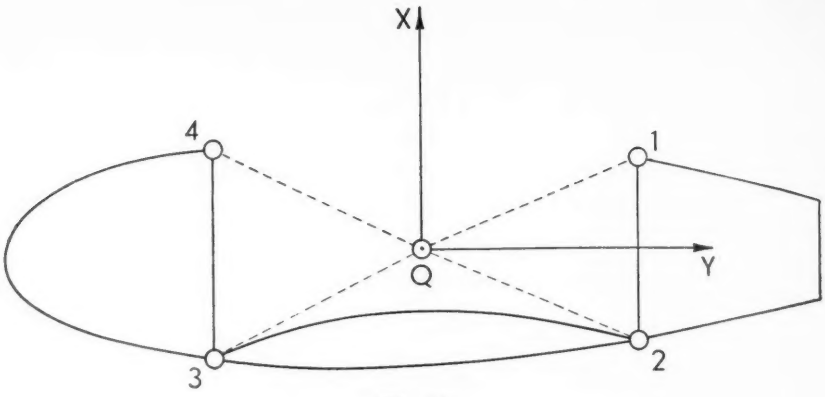


Fig. 34.
Multi-cell four-boom tube with opening.

It is interesting to note that from (537) $J$ is the torsion constant for the original multi-cell open tube.

The above analysis can be immediately generalised to a multi-cell $n$-boom tube with more than one opening. It will also be seen that equation (545) is still valid.

6.4.5. *The h-function for the four-boom multi-cell tube with opening.*

The four-boom tube illustrated in Fig. 34 will now be analysed in detail. The reduction of the multi-cell tube to an equivalent single-cell tube proceeds as described in the previous Section. It is interesting to analyse the equivalent tube in this case by a limiting process from the results of Sections 4.3.2 and 5.6.4.

From equations (192), (539) and (539*b*)

$$\lim \bar{N} = A^*_{23} \qquad (546)$$

and hence,

$$\left.\begin{aligned} \bar{A}_{12} &= A^*_{12} + A^*_{23} = A_{123},\ \bar{A}_{23} = 0 \\ \bar{A}_{34} &= A^*_{34} + A^*_{23} = A_{234},\ \bar{A}_{41} = A^*_{41} - A^*_{23} \end{aligned}\right\} \qquad (547)$$

To obtain $\lambda^2$ one must first find the limit of

$$\delta_{41}(\bar{A}_{23})^2$$

Now from (189),

$$(\bar{A}_{23})^2 = |\ 12,\ 23,\ 34\ |^2/4A^2$$

so that with (539), noting that the missing wall is (41),

$$\lim \delta_{41}(\bar{A}_{23})^2 = (\Xi_{41})^2/(A^m)^2$$

where,

$$\Xi_{41} = \tfrac{1}{2}\ |\ 12,\ 23,\ 34\ | = A_{12}A_{234} - A_{23}(A_{412} - A_{123}) + A_{34}A_{123} \qquad (548)$$

Thus the expression,

$$\Psi = \sum_1^4 \delta_{12}(\bar{A}_{34})^2 \qquad (549)$$

which applies to a single-cell tube or a multi-cell tube without openings becomes in the present case,

$$\Psi = \sum_1^3 \delta_{12}(\bar{A}_{34})^2 + (\Xi_{41}/A^m)^2 \qquad (549a)$$

Hence,

$$\lambda^2=\sum_1^4\frac{(A_{234})^2}{B_1}\Big/\Psi \quad . \quad . \quad . \quad . \quad . \quad . \quad (550)$$

The $h$-functions are

$$h_{r,\,r+1}=(-1)^{r+1}\overline{A}_{\,r+2,\,r+3}/\Psi^{1/2} \quad . \quad . \quad . \quad . \quad (551)$$

where the values of $\overline{A}_{\,r,\,r+1}$ are given in (547).
Also from (198),

$$\Theta=-\Xi_{41}/2\,(A^{\mathrm{m}})^2\,\Psi^{1/2} \quad . \quad . \quad . \quad . \quad . \quad (552)$$

The values of $\xi$ and $\eta$ have to be determined as the limiting forms of equations (389) of Section 5.6.2. Dividing the numerators and denominators of these equations by $\delta_{41}\overline{A}_{23}$ and proceeding to the limit one obtains,

$$\xi=-\frac{\dfrac{A_{234}}{B_1}\,|\,23,\,34\,|_{\mathrm{x}}+\dfrac{A_{341}}{B_2}|\,13,\,34\,|_{\mathrm{x}}+\dfrac{A_{412}}{B_3}\,|\,12,\,24\,|_{\mathrm{x}}+\dfrac{A_{123}}{B_4}\,|\,12,\,23\,|_{\mathrm{x}}}{\sum_1^4\,(A_{234})^2/B_1}-$$

$$-\frac{(A^{\mathrm{m}})^2}{\Xi_{41}}\left\{\delta_{12}A_{234}\left[\frac{A_{412}}{B_3}x_{24}+\frac{A_{123}}{B_4}x_{23}\right]-\delta_{34}A_{123}\left[\frac{A_{234}}{B_1}x_{23}+\frac{A_{341}}{B_2}x_{13}\right]+\right.$$

$$\left.+\delta_{23}\,(A_{341}-A_{234})\left[-\left(\frac{A_{234}}{B_1}+\frac{A_{341}}{B_2}\right)x_{34}+\left(\frac{A_{412}}{B_3}+\frac{A_{123}}{B_4}\right)x_{12}\right]\right\} \quad . \quad (553)$$

and a similar equation for $\eta$ which may be derived from equation (553) by substituting $y$ for $x$. Note that the first expressions on the right hand side of (553) and the corresponding equation for $\eta$ are the co-ordinates $\xi_{\mathrm{ES}}$ and $\eta_{\mathrm{ES}}$ of the actual tube (see equations (619) and Section 6.7.2).

Equation (553) shows immediately that in the present case the centre of zero warping is always at finitum, *i.e.* such tubes as shown in Fig. 34 always warp under torque.

From equations (551) the $L_{\sigma}$, which are identical for the equivalent and actual tube may be obtained. For example,

$$L_{\sigma 1}=A_{234}/\mu B_1\Psi^{1/2} \quad . \quad . \quad . \quad . \quad . \quad (554)$$

The $h^{\circ}$ and $h^{1}$-functions are calculated from (542) and (551) and from those the $L_{\tau}$ functions of the actual tube may be derived. Note that in equations (550)-(554) $\Psi$ is given by equation (549*a*). In concluding the analysis it is emphasised that the quantities occurring in formulæ (549) to (554) all belong to the equivalent tube. Note also that if the opening occurs between two other booms the appropriate formulæ are obtained by cyclic changes and in addition a periodic change of sign occurs in equations (546) and (552).

Some interesting special cases will now be considered. If the sides (12) and (34) of the quadrilateral (1234) are parallel $h_{23}=0$ and the $h$-functions and $\Theta$ are independent of $A_{23}$. Taking the $X$ axis parallel to the sides (12) and (34) $\eta$ is also independent of $A_{23}$, but not $\xi$. Note also that

$$\Xi_{41}=\Xi_{23}=A_{12}A_{234}+A_{34}A_{123} \quad . \quad . \quad . \quad . \quad (548a)$$

If in addition the wall (23) vanishes $\xi$ becomes indeterminate and the tube has no resistance to loads in the $Y$ direction, but there are still axial constraint stresses due to torque about $\eta$.

The case when the wall (23) vanishes while (12) and (34) are not parallel has no eigenloads and the stresses are always in accordance with the engineers' theory under any loading. In place of the eigenload system there is a kinematic freedom to warp associated with a translational movement but no twist (cf. Section 6.5.11). Such a tube can only sustain transverse loads when built-in at all four booms.

### 6.5. THE OPEN TUBE

The main purpose of this Section is to consider the stress distribution and deformation in completely open sections of arbitrary cross-section and under arbitrary loading. In such sections, as occur in wing structures, the torsional stiffness is almost entirely derived from the end constraints, the St. Venant torsional stiffness usually being negligible. The theory to be developed may be regarded as a broad generalisation of the theory of differential bending in two spar wings.

It is convenient to start the investigation of open tubes from the case of an open tube with finite St. Venant torsional stiffness. In such a tube the statically equivalent load system is taken to be the generalised engineers' stresses in bending and the St. Venant shear stresses, the latter being in equilibrium with the torque about the shear centre. The calculation of the eigenloads is facilitated by regarding the St. Venant torsional stiffness to be derived from an imaginary closed tube with walls effective only in shear; the shear stresses in the middle of the walls of the actual tube are unaffected by this analytical device. Thus the investigation is reduced to the analysis of an open tube with no St. Venant torsional stiffness rigidly connected with a closed tube by means of the diaphragms; this case may be treated by the method of 6.4. The $n$-boom open tube with finite St. Venant stiffness is analysed completely.

As stated above, in wing structures the St. Venant stiffness is generally of a negligibly small order, making its retention in the calculations unnecessary and numerically inconvenient. However, the open tube with no St. Venant torsional stiffness is most elegantly analysed as the limiting case of an open tube with a St. Venant stiffness tending to zero. It is shown that the first eigenvalue and eigenfunction vanish and no self-equilibrating end load system can exist in the corresponding mode, but the axial constraint stresses in the latter mode degenerate into the Wagner[23] and Kappus[24], [25] torsion-bending stress system for $J=0$ as generalised for conical tubes. This stress system is not self-equilibrating and will be called hereinafter the $\Gamma$ system; it is statically equivalent to the applied torque about the shear centre. It should be noted that the $\Gamma$ system can only be developed in open tubes, with torque about the shear centre, in which one section at least may be regarded as built-in.

The stresses may now be expressed in the following forms:—

$$\left.\begin{aligned}\sigma &= \sigma_E + \sigma_\Gamma + \sum_i T_{Bio}\Theta_i L_{\sigma i} G_{\sigma i}\\ \tau &= \tau_E + \tau_\Gamma + \sum_i T_{Bio}\Theta_i L_{\tau i} G_{\tau i}\end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad (627)$$

where $\sigma_\Gamma$ and $\tau_\Gamma$ are the stresses of the $\Gamma$ system. Note that the $n$-boom open tube has $n$-4 eigenvalues, *i.e.* one less than the $n$-boom tube with closed cells.

In what follows, unless otherwise stated, the term open tube refers to a tube with $J=0$.

#### 6.5.1. *The open tube with St. Venant torsional stiffness*

Consider an open tube of arbitrary cross-section as shown in Fig. 35. The St. Venant torsion constant $J'$ at $\rho$ is given by,

$$J' = \frac{1}{3}\rho\,(t^*)^3 \int_0^S t^3 ds + \sum_1^n J_m' \quad . \quad . \quad . \quad . \quad (A)$$

where $J_m'$ is the torsion constant of the $m$th boom at $\rho$. In order that the analysis of the present section may be based on the theory of Section 6.4 it is necessary that $J'$ should take the form

$$J' = \rho^3 t^* J \quad . \quad . \quad . \quad . \quad . \quad . \quad (B)$$

where $J$ is the torsion constant at the root.

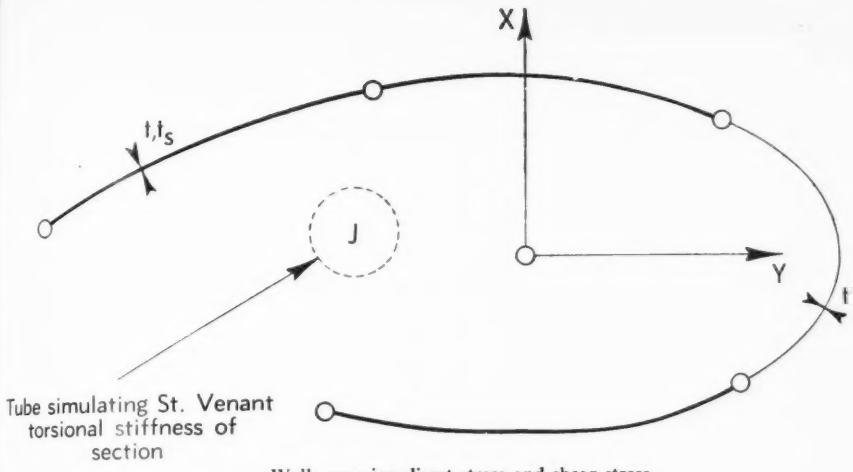


Fig. 35.
Open tube with St. Venant torsion constant $J$.

Equations (A) and (B) are identical only for uniform cylindrical tubes and conical tubes in which all dimensions are proportional to $\rho$. For the solution of the open tube by allowing $J$ to tend to zero the approximation represented by (B) is immaterial; where $J$ is not negligible equation (B) will be in general an approximation.

The torsional stiffness of the open tube is simulated (see Fig. 35) by a closed tube of area $A_J$ and wall effective only in shear with a length/thickness ratio $\delta_J$, such that,

$$J = 4\,(A_J)^2/\delta_J \quad . \quad . \quad . \quad . \quad . \quad (545a)$$

The dimensions of this closed tube must vary as $\rho$ and $t^*$ so that equation (B) is satisfied throughout the tube.

The eigenfunctions will now be investigated on the basis of this idealised structure.

6.5.2. *The eigenfunctions for an open tube with torsion constant J*

The eigenfunctions will be obtained on the lines of Section 6.4. The $h_i$ distribution of the open tube in Fig. 35 must satisfy equation (535)

$$\frac{d}{ds}\left(\frac{1}{t_s}\frac{dh_i}{ds}\right) + \lambda_i^2 \frac{h_i}{t} = \lambda_i^2\, \hat{p}_i\, \Theta_i \quad . \quad . \quad . \quad . \quad (535)$$

The boundary conditions (533) and (533*a*) at the free edges of the open section apply also. For the closed tube equation (534) becomes,

$$\Theta_i = (h_i)_J \delta_J / 2A_J \quad . \quad . \quad . \quad . \quad . \quad (534a)$$

The zero shear conditions in equations (54) noting that the closed tube has no shear resultant, may be written,

$$\int_C h_i \sin\psi\, ds = \int_C h_i \cos\psi\, ds = 0 \quad . \quad . \quad . \quad . \quad (555)$$

where $\int_C$ represents an integration over the open tube only. The zero torque

condition in equations (54) becomes,

$$\oint h_i p ds = \int_C h_i p ds + \oint_J h_i p ds = 0$$

where $\oint_J$ refers to the integration over the closed tube. Using equations (534*a*) and (545*a*) the zero torque condition is,

$$\int_C h_i p ds + J\Theta_i = 0 \qquad . \quad . \quad . \quad . \quad (556)$$

The orthogonality and normalisation conditions (76) and (57) become,

$$\left.\begin{aligned} &\int_C \frac{h_i h_j}{t}\, ds + J\Theta_i\Theta_j = 0, \text{ when } i \neq j \\ &\int_C \frac{h_i^2}{t}\, ds + J\Theta_i^2 = 1 \end{aligned}\right\} \quad . \quad . \quad . \quad (557)$$

From the general solutions of equations (535) which have the form of equation (56), the boundary conditions (62) and (533), and the equilibrium conditions (555) and (556), the secular determinant in $\lambda$ may be obtained (see also the discussion in Section 3.2.1). The roots of the corresponding secular equation yield the eigenvalues $\lambda_i$, while from the ratios of the co-factors of the elements of any row the ratios of,

$$C_{1i},\ C_{2i},\ \ldots\ldots,\ \Theta_i,\ \xi_i\Theta_i,\ \eta_i\Theta_i$$

will be obtained. $\xi_i$ and $\eta_i$ are then known and $C_{1i}, C_{2i}, \ldots\ldots, \Theta_i$ follow from either of the normalising conditions (557) or (78).

The method may easily be extended to a combination of open tubes with $J$ connected only through the same diaphragms.

### 6.5.3. *The n-boom open tube of arbitrary cross-section with torsion constant J*

In this paragraph the $n$-boom open tube of arbitrary cross-section (see Fig. 36) and torsion constant $J$ will be fully worked out.

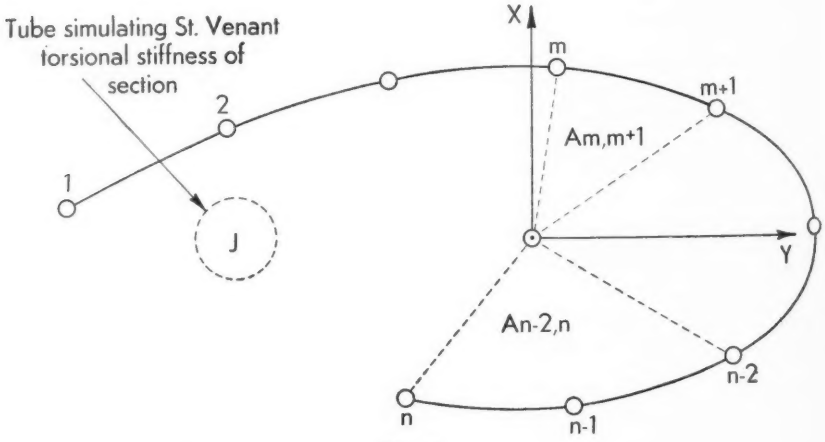


Fig. 36.
Open $n$-boom tube with torsion constant $J$.

Equation (66*a*) is valid in the form,

$$\frac{1}{B_m}h_{m-1,\,m}+\left(\lambda^2\delta_{m,\,m+1}-\frac{1}{B_m}-\frac{1}{B_{m+1}}\right)h_{m,\,m+1}+\frac{1}{B_{m+1}}h_{m+1,\,m+2}$$
$$=\lambda^2\{2A_{m,\,m+1}-\xi y_{m,\,m+1}+\eta x_{m,\,m+1}\}\Theta \quad . \quad . \quad . \quad (558)$$

The zero shear and torque conditions (555) and (556) become,

$$\left.\begin{aligned}\sum_1^{n-1}x_{m,\,m+1}h_{m,\,m+1}=\sum_1^{n-1}y_{m,\,m+1}h_{m,\,m+1}=0\\ 2\sum_1^{n-1}A_{m,\,m+1}h_{m,\,m+1}+J\Theta=0\end{aligned}\right\} \quad . \quad . \quad (559)$$

The normalising condition (557) takes the form,

$$\sum_1^{n-1}\delta_{m,\,m+1}(h_{m,\,m+1})^2+J\Theta^2=1 \quad . \quad . \quad . \quad . \quad (560)$$

From equations (558) and (559) the secular equation may be written as on the following page.

Expanding the determinant of (561) by the method used in Section 5.7.2 for the direct proof of the secular equation (397) one obtains,

$$\lambda^2\Delta_n=\lambda^2\Delta_I-\frac{J}{4}\Delta_{II}=0 \quad . \quad . \quad . \quad . \quad (562)$$

where,

$$\Delta_I=(-1)^{n-1}\sum_{m=0}^{n-4}(-1)^m(\lambda^2)^m\sum_{\binom{n}{m+4}}\left\{\frac{1}{B_rB_s\ldots B_t}\sum_{\binom{m+3}{3}}[\delta_{ij}\delta_{kl}\ldots\delta_{pq}\,|ab,\,cd,\,ef\,|^2]\right\}$$
$$\ldots\ldots\ldots \quad (563)$$

and,

$$\Delta_{II}=(-1)^{n-1}\sum_{m=0}^{n-3}(-1)^m(\lambda^2)^m\sum_{\binom{n}{m+3}}\left\{\frac{1}{B_rB_s\ldots B_t}\sum_{\binom{m+2}{2}}[\delta_{ij}\delta_{kl}\ldots\ldots\delta_{pq}\,|\,cd,\,ef\,|^2]\right\}$$

In these equations . . . (564)

$$r,\,s,\ldots\ldots,\,t\neq i,\quad j,\ldots\ldots,\,q;\quad a,\,b,\ldots\ldots f$$

are boom numbers.

Equation (562) yields $(n-3)$ eigenvalues $\lambda_i$. The process of formation of equations (563) and (564) will now be described.

*I Formation of* $\Delta_I$

The terms,

$$\frac{1}{B_rB_s\ldots\ldots B_t} \quad . \quad . \quad . \quad . \quad . \quad . \quad (400)$$

are formed from all possible products of $(n-m-4)$ different $B$'s selected from the $n$-booms; thus there are in all $\binom{n}{m+4}$ such terms for the $m$th power of $\lambda^2$. It should be noted that the coefficient of the highest power of $\lambda^2$ does not involve any $B$.

To form the summation,

$$\sum_{\binom{m+3}{3}}[\delta_{ij}\delta_{kl}\ldots\ldots\delta_{pq}\,|\,ab,\,cd,\,ef\,|^2] \quad . \quad . \quad . \quad . \quad (401)$$

under the terms (400) one considers the open tube with $(m+4)$ booms obtained by omitting the $(n-m-4)$ booms $B_r, B_s, \ldots\ldots, B_t$ from the original $n$-boom open tube and joining the remaining booms. In the open tube so formed the booms are joined by $(m+3)$ walls,

$$ij,\,kl,\ldots\ldots,\,pq,\,ab,\,cd,\,ef$$

which yield the $\binom{m+3}{3}$ products appearing in (401).

$$\Delta_n = \begin{vmatrix}
\left[\lambda^2\delta_{12} - \frac{1}{B_1} - \frac{1}{B_2}\right] & \frac{1}{B_2} & 0 & 0 & 0 & 0 & A_{12} & x_{12} & y_{12} \\
\frac{1}{B_2} & \left[\lambda^2\delta_{23} - \frac{1}{B_2} - \frac{1}{B_3}\right] & \frac{1}{B_3} & 0 & 0 & 0 & A_{23} & x_{23} & y_{23} \\
| & | & | & | & | & | & | & | \\
| & | & | & | & | & | & | & | \\
| & | & | & | & | & | & | & | \\
| & | & | & | & | & | & | & | \\
0 & 0 & \frac{1}{B_m} & \left[\lambda^2\delta_{m,\,m+1} - \frac{1}{B_m} - \frac{1}{B_{m+1}}\right] & \frac{1}{B_{m+1}} & 0 & A_{m,\,m+1} & x_{m,\,m+1} & y_{m,\,m+1} \\
| & | & | & | & | & | & | & | \\
| & | & | & | & | & | & | & | \\
| & | & | & | & | & | & | & | \\
| & | & | & | & | & | & | & | \\
0 & 0 & 0 & \frac{1}{B_{n-2}} & \left[\lambda^2\delta_{n-2,\,n-1} - \frac{1}{B_{n-2}} - \frac{1}{B_{n-1}}\right] & \frac{1}{B_{n-1}} & A_{n-2,\,n-1} & x_{n-2,\,n-1} & y_{n-2,\,n-1} \\
0 & 0 & 0 & 0 & \frac{1}{B_{n-1}} & \left[\lambda^2\delta_{n-1,\,n} - \frac{1}{B_{n-1}} - \frac{1}{B_n}\right] & A_{n-1,\,n} & x_{n-1,\,n} & y_{n-1,\,n} \\
A_{12} & A_{23} & \cdots & A_{m,\,m+1} & A_{n-2,\,n-1} & A_{n-1,\,n} & -\frac{J}{4\lambda^2} & 0 & 0 \\
x_{12} & x_{23} & \cdots & x_{m,\,m+1} & x_{n-2,\,n-1} & x_{n-1,\,n} & 0 & 0 & 0 \\
y_{12} & y_{23} & \cdots & y_{m,\,m+1} & y_{n-2,\,n-1} & y_{n-1,\,n} & 0 & 0 & 0
\end{vmatrix} = 0$$

. . . . . (561)

*II Formation of* $\Delta_{II}$

The terms,

$$\frac{1}{B_r B_s \ldots\ldots B_t} \quad . \quad . \quad . \quad . \quad . \quad . \quad (400)$$

are found from all possible products of $(n-m-3)$ different $B$'s selected from the $n$-booms; thus there are in all $\binom{n}{m+3}$ such terms for the $m$th power of $\lambda^2$.

To form the summation,

$$\sum_{\binom{m+2}{2}} [\delta_{ij}\delta_{kl} \ldots\ldots \delta_{pq} \,|\, cd,\ ef\,|^2] \quad . \quad . \quad . \quad . \quad (565)$$

under the terms (400) one considers the open tube with $(m+3)$ booms obtained by omitting the $(n-m-3)$ booms $B_r$, $B_s$, $\ldots\ldots B_t$ from the original $n$-boom open tube and joining the remaining booms. In the open tube so formed the booms are joined by $(m+2)$ walls,

$$ij,\ kl,\ \ldots\ldots,\ pq,\ cd,\ ef$$

which yield the $\binom{m+2}{2}$ products appearing in (565).

Denoting the co-factors of the elements in the $n$th row of the determinant of (561) by,

$$\Omega_1, \Omega_2, \ldots\ldots, \Omega_R, \ldots\ldots, \Omega_{n+2}$$

one has,

$$\frac{h_{12}}{\Omega_1} = \frac{h_{23}}{\Omega_2} = \ldots\ldots = \frac{h_{R,\,R+1}}{\Omega_R} = \ldots\ldots = \frac{h_{n-1,\,n}}{\Omega_{n-1}} = \frac{-2\lambda^2\Theta}{\Omega_n}$$

$$= \frac{-\lambda^2\eta\Theta}{\Omega_{n+1}} = \frac{\lambda^2\xi\Theta}{\Omega_{n+2}} = \text{const.} \quad . \quad . \quad (566)$$

By the methods of Section 5.7.3 for $R=1$ to $n-1$,

$$\Omega_R = \frac{1}{2}\frac{\partial\Delta_I}{\partial A_{R,\,R+1}}$$

$$= (-1)^{n-1} \sum_{0}^{n-4} (-1)^m (\lambda^2)^m \sum_{\kappa} \left\{ \frac{1}{B_r B_s \ldots B_t} \sum_{\binom{m+2}{2}} [\delta_{ij}\delta_{kl} \ldots \delta_{pq} \,|\, ab,\ cd,\ ef\,| \,.\, |\, cd,\ ef\,|] \right\} \quad (567)$$

where

$$\kappa = \binom{n}{m+4} - \binom{R}{m+4} - \binom{n-R}{m+4}$$

is the total number of terms in $(\lambda^2)^m$ and $ab$ must always include the wall, $R$, $R+1$. The dependence of $\kappa$ on $R$ is explained by the fact that in forming the terms (400) with $(n-m-4)$ $B$'s the remaining $(m+4)$ boom open tube *must* contain the wall $R$, $R+1$.

Note that there are in all $\binom{m+2}{2}$ terms of the form

$$\delta_{ij}\delta_{kl} \ldots\ldots \delta_{pq} \,|\, ab,\ cd,\ ef\,| \;\; |\, cd,\ ef\,| \quad . \quad . \quad . \quad (415)$$

for each term (400).

The co-factor $\Omega_n$ may be obtained similarly. Thus,

$$\Omega_n = \frac{\partial(\lambda^2\Delta_n)}{\partial(-J/4)} = \Delta_{II} \quad . \quad . \quad . \quad . \quad . \quad (568)$$

For the co-factors $\Omega_{n+1}$, $\Omega_{n+2}$ the method used to prove directly the secular equation (397), yields,

$$\Omega_{n+1} = (-1)^n \sum_{m=0}^{n-3} (-1)^m (\lambda^2)^m \sum_{\binom{n}{m+3}} \left\{ \frac{1}{B_r B_s \ldots B_t} \sum_{\binom{m+2}{2}} [\delta_{ij}\delta_{kl} \ldots \delta_{pq} \,|\, cd,\ ef\,| \;|\; cd,\ ef\,|_\gamma] \right\} \quad (569)$$

and,

$$\Omega_{n+2}=(-1)^{n-1}\sum_{m=0}^{n-3}(-1)^m(\lambda^2)^m\sum_{\binom{n}{m+3}}\left\{\frac{1}{B_rB_s\ldots B_t}\sum_{\binom{m+2}{2}}[\delta_{ij}\delta_{kl}\ldots\delta_{pq}\,|\,cd,\,ef\,|\,|\,cd,\,ef\,|_x]\right\} \quad (570)$$

Note that in these equations there are $(n-m-3)$ $(1/B)$'s and $m$ $\delta$'s in each term under $(\lambda^2)^m$.

From equations (566) and the normalising condition (560),

$$h_{R,\,R+1}=\frac{2\lambda^2\Omega_R}{[4\lambda^4\sum_1^{n-1}\delta_{P,\,P+1}(\Omega_P)^2+J\,(\Omega_n)^2]^{1/2}} \quad . \quad . \quad . \quad (571)$$

and,

$$\Theta=-\frac{\Omega_n}{[4\lambda^4\sum_1^{n-1}\delta_{P,\,P+1}(\Omega_P)^2+J\,(\Omega_n)^2]^{1/2}} \quad . \quad . \quad . \quad (572)$$

Also,

$$\left.\begin{aligned}\xi&=-\frac{2\Omega_{n+2}}{\Omega_n}\\ \eta&=\frac{2\Omega_{n+1}}{\Omega_n}\end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (573)$$

In the exceptional case when $\Theta=0$, $\Omega_n=0$, and one obtains

$$h_{R,\,R+1}=\frac{\Omega_R}{[\sum_1^{n-1}\delta_{P,\,P+1}(\Omega_P)^2]^{1/2}} \quad . \quad . \quad . \quad . \quad (571a)$$

and

$$\left.\begin{aligned}b_x=\xi\Theta&=\frac{\Omega_{n+2}}{\lambda^2[\sum_1^{n-1}\delta_{P,\,P+1}(\Omega_P)^2]^{1/2}}\\ b_y=\eta\Theta&=-\frac{\Omega_{n+1}}{\lambda^2[\sum_1^{n-1}\delta_{P,\,P+1}(\Omega_P)^2]^{1/2}}\end{aligned}\right\} \quad . \quad . \quad . \quad . \quad (573a)$$

The $L$-functions as defined by equation (340) can now be obtained from equations (571) and (571*a*). The condition that $\Theta_i$ is zero will be satisfied if $\lambda_i$ is a common root of

$$\Delta_I=0 \text{ and } \Delta_{II}=0.$$

The results of this paragraph may be applied to the analysis of an $n$-boom multi-cell tube with openings as shown in Section 6.4.4. On the other hand the particular case of a four-boom open tube with $J$ may be obtained from Section 6.4.5 by substituting $J$ for $4\,(A^m)^2$.

### 6.5.4. *The eigenfunctions for a symmetrical multi-boom open tube with torsion constant J*

When a multi-boom open tube possesses an axis of symmetry a considerable simplification of the results given in Section 6.5.3 is possible.

An open tube symmetrical about the $Y$-axis with $2n$ booms is illustrated in Fig. 37. No boom need be placed at $\beta$ on the axis of symmetry since in such a case the appropriate results may be obtained by coalescence of the pair of booms $n$ and $n+1$ so that,

$$\delta_{n,\,n+1}=0.$$

Note that the relations (428) given in Section 5.7.5 for a singly symmetrical tube apply also here.

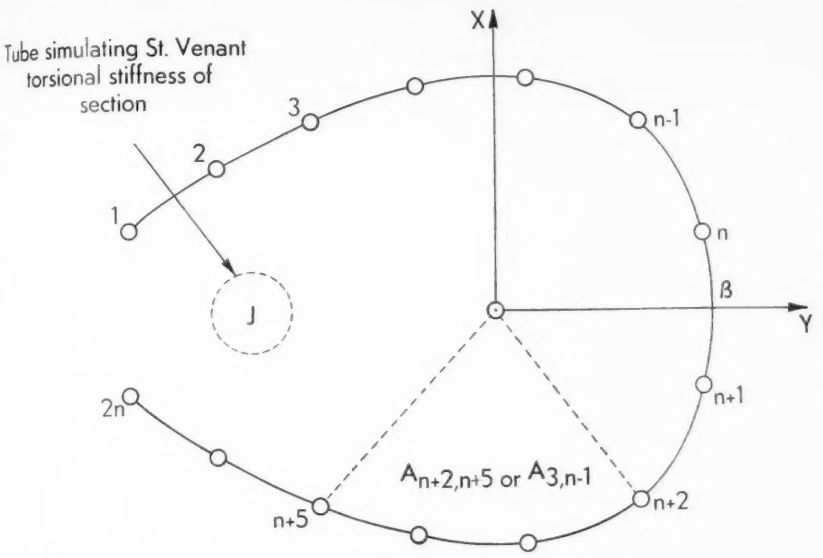


Fig. 37.
Open symmetrical multi-boom tube with torsion constant $J$.

Exactly as in Section 5.7.5 the symmetry separates the eigenfunctions into two distinct groups, namely:—

(*a*) eigenfunctions involving flexure in the $YZ$-plane for which relations (429) are satisfied, and

(*b*) eigenfunctions involving twist and/or flexure in the $XZ$-plane for which relations (430) are satisfied.

These two cases will be separately considered.

(*a*) *The eigenfunctions for flexure in the YZ-plane*

The torsion constant $J$ does not enter in these modes and the results are exactly as given in Section 5.7.5 (*a*) for the eigenfunctions for flexure in the plane of symmetry of a singly symmetrical closed tube. The value of $b_x$ is obtained from equation (442). Thus,

$$b_x = \frac{\Omega_n}{\lambda^2 \left[2 \sum_{1}^{n-1} \delta_{P,\,P+1} (\Omega_P)^2\right]^{1/2}} \quad . \quad . \quad . \quad . \quad (442a)$$

(*b*) *The eigenfunctions for twist and flexure in the XZ-plane*

For these eigenfunctions by virtue of equations (428) and (430),

$$\sum_{1}^{2n-1} y_{r,\,r+1}\, h_{r,\,r+1} = 0 \quad . \quad . \quad . \quad . \quad . \quad (574)$$

so that the last of equations (558) is automatically satisfied.

Thus with the notation,

$$\delta_{n\beta} = \frac{1}{2}\delta_{n,\,n+1}, \quad A_{n\beta} = \frac{1}{2} A_{n,\,n+1} \quad . \quad . \quad . \quad . \quad (575)$$

$$\Delta_{XT}=\begin{vmatrix}
\left[\lambda^2\delta_{12}-\frac{1}{B_1}-\frac{1}{B_2}\right] & \frac{1}{B_3} & 0 & 0 & 0 & 0 & A_{12} & x_{12}\\
\frac{1}{B_2} & \left[\lambda^2\delta_{23}-\frac{1}{B_2}-\frac{1}{B_3}\right] & \frac{1}{B_3} & 0 & 0 & 0 & A_{23} & x_{23}\\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots\\
0 & 0 & \frac{1}{B_m} & \left[\lambda^2\delta_{m,m+1}-\frac{1}{B_m}-\frac{1}{B_{m+1}}\right] & \frac{1}{B_{m+1}} & 0 & A_{m,m+1} & x_{m,m+1}\\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots\\
0 & 0 & 0 & 0 & \left[\lambda^2\delta_{n-1,n}-\frac{1}{B_{n-1}}-\frac{1}{B_n}\right] & \frac{1}{B_n} & A_{n-1,n} & x_{n-1,n}\\
0 & 0 & 0 & 0 & \frac{1}{B_n} & \left[\lambda^2\delta_{n\beta}-\frac{1}{B_n}\right] & A_{n\beta} & x_{n\beta}\\
A_{12} & A_{23} & \cdots & A_{m,m+1} & A_{n-1,n} & A_{n\beta} & -\frac{J}{8\lambda^2} & 0\\
x_{12} & x_{23} & \cdots & x_{m,m+1} & x_{n-1,n} & x_{n\beta} & 0 & 0
\end{vmatrix}=0 \qquad (576)$$

the secular equation becomes as on the following page. Expanding this determinant one obtains,

$$\lambda^2\Delta_{XT}=\lambda^2\bar{\Delta}_I-\frac{J}{8}\bar{\Delta}_{II}=0 \quad . \quad . \quad . \quad . \quad . \quad (577)$$

where,

$$\bar{\Delta}_I=(-1)^n\sum_0^{n-2}(-1)^m(\lambda^2)^m\sum_{\binom{n}{m+2}}\left\{\frac{1}{B_rB_s\ldots\ldots B_t}\sum_{\binom{m+2}{2}}[\delta_{ij}\delta_{kl}\ldots\ldots\delta_{pq}\mid ab,\ cd\mid_x^2]\right\} \quad (578)$$

and

$$\bar{\Delta}_{II}=(-1)^n\sum_0^{n-1}(-1)^m(\lambda^2)^m\sum_{\binom{n}{m+1}}\left\{\frac{1}{B_rB_s\ldots\ldots B_t}\sum_{m+1}[\delta_{ij}\delta_{kl}\ldots\ldots\delta_{pq}(x_{ab})^2]\right\} \quad (579)$$

where $r, s, \ldots\ldots, t$ are boom numbers and $i, j, \ldots\ldots q$; $a, b, c, d$ which are different from $r, s, \ldots\ldots, t$ are boom numbers and the point $\beta$. Equation (577) yields $(n-1)$ eigenvalues $\lambda_i$. These roots together with the $(n-2)$ roots of case $(a)$ make the necessary total of $2n-3$ roots.

The process of formation of equations (578) and (579) will now be described.

*I Formation of* $\bar{\Delta}_I$

The terms,

$$\frac{1}{B_rB_s\ldots\ldots B_t} \quad . \quad . \quad . \quad . \quad . \quad (400)$$

are formed from all possible products of $(n-m-2)$ different $B$'s selected from the $n$-booms; thus there are in all $\binom{n}{m+2}$ such terms for the $m$th power of $\lambda^2$.

To form the summation,

$$\sum_{\binom{m+2}{2}}[\delta_{ij}\delta_{kl}\ldots\ldots\delta_{pq}\mid ab,\ cd\mid_x^2] \quad . \quad . \quad . \quad . \quad (446)$$

under the terms (400) one considers the open tube with $(m+2)$ booms obtained by omitting the $(n-m-2)$ booms $B_r, B_s, \ldots\ldots, B_t$ from the original tube and joining the remaining booms and the end $\beta$. In the open tube so formed the booms and the point $\beta$ are joined by $(m+2)$ walls

$$ij,\ kl,\ \ldots\ldots,\ pq,\ ab,\ cd,$$

which yield the $\binom{m+2}{2}$ products appearing in (446).

*II Formation of* $\bar{\Delta}_{II}$

The law of formation of $\bar{\Delta}_{II}$ is similar to that of $\bar{\Delta}_I$. Note that there are $(n-m-1)\frac{1}{B}$'s and $m$ $\delta$'s in each term under $(\lambda^2)^m$.

Denoting the co-factors of the elements in the $(n+1)$th row of the determinant of (576) by,

$$\Omega_1, \Omega_2, \ldots\ldots, \Omega_R, \ldots\ldots, \Omega_n, \Omega_{n+1}, \Omega_{n+2}$$

one has,

$$\frac{h_{12}}{\Omega_1}=\frac{h_{23}}{\Omega_2}=\ldots\ldots=\frac{h_{R,R+1}}{\Omega_R}=\ldots\ldots=\frac{h_{n\beta}}{\Omega_n}=\frac{-2\lambda^2\Theta}{\Omega_{n+1}}=\frac{-\lambda^2\eta\Theta}{\Omega_{n+2}}=\text{const.} \quad (580)$$

By the methods of Section 5.7.3 for $R=1$ to $n$,

$$\Omega_R=\frac{1}{2}\frac{\partial\bar{\Delta}_I}{\partial A_{R,\,R+1}}$$

$$=(-1)^n\sum_0^{n-2}(-1)^m(\lambda^2)^m\sum_\kappa\left\{\frac{1}{B_rB_s\ldots\ldots B_t}\sum_{m+1}[\delta_{ij}\delta_{kl}\ldots\ldots\delta_{pq}\mid ab,\,cd\mid x_{cd}]\right\}\quad(581)$$

$$\text{where }\kappa=\binom{n}{m+2}-\binom{n-R}{m+2}$$

is the total number of terms in $(\lambda^2)^m$ and $ab$ must always include the wall $R$, $R+1$. The dependence of $\kappa$ on $R$ is explained by the fact that in forming the terms (400) with $(n-m-2)$ $B$'s the combinations are excluded which contain $B_1$ to $B_R$ inclusive. The co-factor $\Omega_{n+1}$ may be obtained similarly.

Thus,

$$\Omega_{n+1}=\frac{\partial\Delta_{XT}}{\partial(-J/8)}=\bar{\Delta}_{II}\qquad(582)$$

For the co-factor $\Omega_{n+2}$ the method used to prove directly the secular equation (397) yields,

$$\Omega_{n+2}=(-1)^{n-1}\sum_0^{n-1}(-1)^m(\lambda^2)^m\sum_{\binom{n}{m+1}}\left\{\frac{1}{B_rB_s\ldots\ldots B_t}\sum_{m+1}[\delta_{ij}\delta_{kl}\ldots\ldots\delta_{pq}A_{ab}x_{ab}]\right\}\quad(583)$$

The rule of formation of (583) follows closely that of $\bar{\Delta}_{II}$.

From equation (581) and the normalising condition (560),

$$h_{R,\,R+1}=\frac{2\lambda^2\Omega_R}{[8\lambda^4\sum_1^n\delta_{P,\,P+1}(\Omega_P)^2+J(\Omega_{n+1})^2]^{1/2}}\qquad(584)$$

and,

$$\Theta=-\frac{\Omega_{n+1}}{[8\lambda^4\sum_1^n\delta_{P,\,P+1}(\Omega_P)^2+J(\Omega_{n+1})^2]^{1/2}}\qquad(585)$$

Also

$$\eta=\frac{2\Omega_{n+2}}{\Omega_{n+1}}\qquad(586)$$

The exceptional case when $\Theta_1=0$ may be treated as in Section 6.5.3.

### 6.5.5. *The stresses and deformations for an open tube with torsion constant J*

The differential equation (299) for the $g_1$-function, and the expressions for the stresses and deformations are also applicable here with some obvious modifications. Thus where

$$\frac{1}{2A}\oint_s\frac{h_1}{t}\,ds$$

occurs substitute $\Theta_1$.

For example the differential equation (299) becomes,

$$\frac{d}{dr}\left(\frac{\rho}{t^*}\frac{dg_1}{dr}\right)-\mu_1{}^2\frac{g_1}{\rho t_s{}^*}=\Theta_1\frac{d}{dr}\left(\frac{T_{B1}}{\rho t^*}\right)\qquad(299b)$$

The torque for the St. Venant shear stresses is always referred to the shear centre $S$ so that $T_{BE}$ becomes $T_{BS}$. The calculation of $h_X$ and $h_Y$ is straightforward since the

origin of the integration in the second of equations (275) is clearly at one edge of the tube. Finally the expression in the twist

$$\frac{1}{4A^2G}\oint_S \frac{h_B}{t}\,ds$$

becomes by virtue of equation (552),

$$\frac{1}{GJ}$$

With these substitutions the stresses and deformations become as follows:— From equation (314) the stresses are,

$$\left.\begin{aligned}\sigma &= \sigma_E + \sum_i T_{Bio}\Theta_i G_{\sigma i} L_{\sigma i}\\ \tau &= \tau_E + \sum_i T_{Bio}\Theta_i G_{\tau i} L_{\tau i}\end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad (587)$$

where the $G$-functions are the same as in Parts IV and V and are defined by,

$$\left.\begin{aligned}G_{\sigma i} &= \frac{g_i \mu_i}{\rho t_s^*} \Big/ T_{Bio}\Theta_i\\ G_{\tau i} &= -\frac{1}{t^*}\frac{dg_i}{dr} \Big/ T_{Bio}\Theta_i\end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad (315a)$$

In addition there are, of course, the St. Venant shear stresses due to $T_{BS}$.

From equations (353), (359) and (360) the deformations for a tube built-in at $r=r_o$ and free at $r=r_e$ are

$$\theta = -\frac{1}{GJ}\int_r^{r_o}\frac{T_{BS}}{\rho^3 t^*}\,dr - \frac{1}{G}\sum_i T_{Bio}\Theta_i^2 \int_r^{r_o}\frac{G_{\tau i}}{\rho}\,dr \quad . \quad . \quad . \quad (588)$$

$$u_S = -\frac{1}{EI_y}\int_{r_o}^{r}\int_{r_o}^{r}\frac{M_y}{\rho^2 t_s^*}\,dr\,dr + \frac{1}{GS^2}\int_C \frac{h_X^2}{t}\,ds \,.\, \rho\int_{r_o}^{r}\frac{Q_x}{\rho^2 t^*}\,dr + \frac{1}{GS^2}\int_C \frac{h_X h_Y}{t}\,ds \,.\, \rho\int_{r_o}^{r}\frac{Q_y}{\rho^2 t^*}\,dr +$$

$$+ \frac{1}{G}\sum_i \left\{ T_{Bio}\Theta_i^2 (\eta_i - \eta_{ES})\,\rho \int_{r_o}^{r}\frac{G_{\tau i}}{\rho}\,dr \right\} \quad . \quad . \quad . \quad (589)$$

The formula for $v_S$ is similar. The deformations for the cylindrical tube may be written down at once from the above equation.

For modes in which $\Theta_i$ is zero while $S_i$ is at infinity, as would occur for example in the flexural modes of a symmetrical cross section, one writes,

$$(Q_{xo}b_{yi} - Q_{yo}b_{xi}) \text{ for } T_{Bio}\Theta_i$$

and

$$(Q_{xo}b_{yi} - Q_{yo}b_{xi})\,b_{yi} \quad \text{for} \quad T_{Bio}\Theta_i^2 (\eta_i - \eta_{ES})$$

The warping function follows from equation (365) and is,

$$W_i^* = \frac{\Theta_i}{G}\left[\frac{T_{Bi}}{\rho t^*} + T_{Bio}\rho G_{\tau i}\right] \quad . \quad . \quad . \quad . \quad (590)$$

or when $\Theta_i = 0$,

$$W_i^* = \frac{1}{G}\left[\frac{1}{t^*}(Q_x b_{yi} - Q_y b_{xi}) + (Q_{xo}b_{yi} - Q_{yo}b_{xi})\,\rho G_{\tau i}\right] \quad . \quad . \quad (590a)$$

*(End of Section 6.55.*

*To be concluded.)*

# REVIEWS

**Military and Political Consequences of Atomic Energy.** P. M. S. Blackett, Turnstile Press, London. 1948. Third Impression (Revised) February 1949. 222 pp. 12s. 6d. net.

This is an important book, a specious book, and a dangerous book.

It is important by reason of its subject.

It is specious by reason of the training of its author which has enabled him to put forward his own emotional opinions under the guise of pseudo-scientific factual statements.

It is dangerous because Professor Blackett is a most distinguished experimental physicist whose opinions, let alone arguments, may be taken as facts by those who are blinded by his admitted brilliance in the cosmic field.

In Book XXXV of his Natural History, Pliny the Elder told a story nearly 2,000 years ago, of the famous Greek painter Apelles who used to conceal himself behind his pictures and listen to the criticisms made on them. One day he was censured by a shoemaker for having represented the shoes with one latchet too few. The next day, the shoemaker, quite proud at seeing the error corrected, thanks to his advice, began to criticise the leg; upon which Apelles, full of indignation, popped his head out and reminded him that a shoemaker should give no opinion beyond the shoes—a piece of advice which has equally passed into a proverbial saying.

While Professor Blackett sticks to his last of physics there are few who can dispute with him on equal terms. But when he voices his opinions in the field of military endeavour he should remember that the military mind is essentially an offensive one, whether it is Russian or American.

Professor Blackett might also recall Hamlet speaking of the skull dug up by the first clown in the churchyard, when he enters the field of politics.

" That skull had a tongue in it, and could sing once . . . . It might be the pate of a politician . . . . one that would circumvent God, might it not? "

To bolster up his military and political arguments Professor Blackett makes use of evidence which he would scorn to accept in the field of cosmic rays. What Alexander Werth correspondent of the *Manchester Guardian* in Moscow, says of political opinion in Russia is evidence of what Russia probably thinks! The correspondent would not stay alive long in Moscow if it were evidence! A scare headline and paragraphs in the *New York Post* suggest the United States were near sending atomic bombs to Germany during the Jugoslavia crisis of 1946.

Of the so-named Baruch plan for the control of atomic energy, which had for its chief ingredient the Lilienthal plan, the creation of the scientists themselves, Blackett remarks " What was intended by the atomic scientists to bring co-operation with Russia became an instrument in the hands of the American Government to coerce her . . . . In short the atomic scientists were out-manœuvred for a second time."

In the eyes of Professor Blackett the United States' politicians are certainly the cat among the Russian pigeons. Nor is it likely, even if they had all they wanted, that the Russians would drop atomic bombs on large cities, says the author. Professor Blackett has his solution for all the troubles which seem to be facing the world. It is world government.

" Though it is, of course, impossible to predict the form that any system of world government might take in the future, or how it will come into being, it is easy to predict that it will not begin with world government in one weapon and one commodity, what ever the essence of the majority report of the Atomic Energy Commission of the United Nations."

Of course the minority was the U.S.S.R. and of course the majority failed to "appreciate how the early stages of the plan would affect Soviet military security."

"More generally," adds Blackett, "the whole conception of the possibility of a *fair and equitable* allocation of such a potentially important economic factor as atomic power by an impartial international body is surely illusory in a world where such great differences exist in the natural resources, degree of industrialisation and standard of life of the various nations." (The italics are the author's.)

It was said at the beginning of this review that the book was specious. The word is defined in the Oxford Dictionary as "plausible, fair or right on the surface."

And that is exactly what Professor Blackett has achieved in his book.

He puts too much faith in the humane nature of the human being.

Especially if it speaks Russian.

**Air Transport Navigation:** P. H. Redpath and J. M. Coburn. Pitman Publishing Corporation, New York. 1949 (first published 1943) (3rd printing). 612 pp. Diagrams, Index. 25s. net.

"Navigation in American internal airlines" would have made a better title for this book, which provides an exhaustive description of navigation technique in the land of radio ranges.

Mr. Redpath is T.W.A.'s chief navigation officer and he has added to the literature of air navigation an authentic picture of its practice inside the U.S.A. in 1943, when it appears that drift meters were just being introduced and the prospect of V.H.F. radio ranges spaced at 100 mile intervals along the air routes promised a navigator's Eldorado.

The 600 pages are about equally divided between treatments of dead-reckoning and radio range techniques and are intended for pilots and navigators. Meteorology has been omitted but there are 11 pages on astro navigation "to whet the reader's appetite."

Chapters 4 and 5, which are devoted to aircraft instruments, have been rendered out-of-date by developments during the war. Remote indicating and gyro-magnetic compasses are mentioned only as possible propositions and automatic D.R. is not suggested, even as a pipe dream. Oddly enough, however, the British drift recorder is named as runner-up to the Pioneer B.3.

In these sections particularly, the preface's claim to supply "theoretical proof for the sceptic" is hardly justified. "Areas of longitude" on p. 35 might be a slip of the pen, but the persistent representation of the magnetic pole by a horse-shoe magnet, an analogy which is repeated in the preamble to the compass-swinging description on p. 67, is not calculated to inspire confidence in the authors' soundness. Nor does the statement on p. 86 that the air density "changes in a direct relation to the product of the pressure and temperature."

The ensuing five chapters deal with simple D.R. practice, culminating in the three drift wind and radius of action problems. Eight chapters on radio range flying then give a realistic impression and some practical hints on American methods.

A list of the shortcomings of this book would be tedious—sufficient to say that it is superficial and falls lamentably short of the classic "Air Navigation" by Redpath's compatriot Lt.-Cdr. Weems.

Mr. Redpath has adopted an informal style, he tells us, to denote the practical usefulness of navigation but it is not, to use his word, overly easy to read, particularly as he consistently rambles and repeats himself.

# CORRESPONDENCE

## AIRCRAFT ACCIDENTS

Referring to my letter regarding aircraft accidents in the November 1948 JOURNAL, I wish to reply to Squadron Leader H. C. Brilliant's letter in the March 1949 JOURNAL.

*Ground control*

What Squadron Leader Brilliant says is quite true, but even the pilot of a ship issues navigation instructions to the appropriate members of the crew, while, under the " talk you down " system, the ground controller issues similar instructions to the aircraft and has, therefore, some degree of control.

The point I wished to make was that, although the ground controller may transmit vital information to the aircraft, the pilot is not compelled to act upon it. If the controller, for example, diverts an aircraft to another aerodrome where landing conditions are safer, the pilot may either go there, choose his own alternative, or he may still attempt to get down, and this state of affairs does not, to my mind, promote safety.

*Passengers and safety equipment*

With regard to the main cabin door I feel that passengers would be more likely to panic in the event of fire than in the ditching case. In the former, emergency exits would most likely be forgotten and the time factor is short. In the latter, aircraft invariably ditch in the dose-down attitude and the cabin door may be above the water line. If water covered, or partially covered the door, no attempt should be made to open it as certain emergency exits are arranged to be above water, particularly in current designs.

In my original correspondence (November 1948 JOURNAL) under the heading of " Crash Fires," item 4, the last sentence should read " In this connection pipelines should not be run on the front face of spars." (The word " front " was inadvertently omitted—*Ed.*)

**H. Smith**

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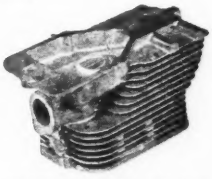

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THE ROYAL AERONAUTICAL SOCIETY
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# NOTICES MAY 1949

## WHITSUN HOLIDAYS

The Offices and Library of the Society will be closed from 5 p.m. on Friday, 3rd June to 9 a.m. on Tuesday, 7th June 1949 for the Whitsun Holidays.

## CONTENTS OF THE MAY JOURNAL

The Modern Auto-Pilot, by F. W. Meredith, B.A., F.R.Ae.S.
Radar as an Aid to the Study of the Atmosphere, by F. E. Jones, M.B.E., B.Sc., Ph.D., A.M.I.E.E.
The Design and Development of the Helicopter Rotor Testing Tower, by R. F. Tayler, A.M.I.Mech.E.
The General Theory of Cylindrical and Conical Tubes under Torsion and Bending Loads—Part VI, by J. Hadji-Argyris, D.E., A.F.R.Ae.S., and P. C. Dunne, B.Sc., A.F.R.Ae.S.
Reviews.
Correspondence.

*The Council have set aside an annual sum of £250 for the award of premiums for papers published in the Journal and hope that members (or non-members) will contribute papers on their own special subjects.*

## ELECTION OF NEW PRESIDENT

Sir John S. Buchanan, Vice-President, was elected President of the Society for the year 1949-50 at the Council Meeting on 31st March 1949.

## ELECTION OF COUNCIL MEMBERS FOR 1949-52

As a result of the ballot declared at the Annual General Meeting on 6th May 1949 the following were elected to fill the vacancies on Council for 1949-52:—

G. R. Edwards, Esq., M.B.E., B.Sc., F.R.Ae.S., A.M.I.Struct.E.
Sir A. H. Roy Fedden, M.B.E., F.R.S.A., D.Sc., F.R.Ae.S., M.I.Mech.E., M.S.A.E., Hon.F.I.Ae.S.
Sir Arthur Gouge, B.Sc., F.R.Ae.S.
Major F. B. Halford, C.B.E., F.R.Ae.S.
W. G. A. Perring, Esq., F.R.Ae.S.
N. E. Rowe, Esq., C.B.E., B.Sc., D.I.C., F.R.Ae.S.
Captain C. F. Uwins, A.F.C., O.B.E., F.R.Ae.S.

The ordinary members of Council for the year 1949-50 are, therefore:—

Lord Brabazon of Tara, M.C., F.R.Ae.S.
Major G. P. Bulman, C.B.E., B.Sc., F.R.Ae.S.
S. Camm, Esq., C.B.E., F.R.Ae.S.
A. V. Cleaver, Esq., A.R.Ae.S.
Dr. G. P. Douglas, O.B.E., M.C., F.R.Ae.S.
G. R. Edwards, Esq., M.B.E., B.Sc., F.R.Ae.S., A.M.Struct.E.
A. G. Elliott, Esq., C.B.E., M.S.A.E., F.R.Ae.S.
W. S. Farren, Esq., C.B.E., F.R.S., F.R.Ae.S.
Sir A. H. Roy Fedden, M.B.E., F.R.S.A., D.Sc., F.R.Ae.S., M.I.Mech.E., M.S.A.E., Hon.F.I.Ae.S.
Sir Arthur Gouge, B.Sc., F.R.Ae.S.

Major F. B. Halford, C.B.E., F.R.Ae.S.
Professor A. A. Hall, M.A., F.R.Ae.S.
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W. Tye, Esq., B.Sc., F.R.Ae.S.
Captain C. F. Uwins, A.F.C., O.B.E., F.R.Ae.S.
Dr. H. C. Watts, M.B.E., M.Inst.C.E., F.R.Ae.S.

## HONORARY FELLOWSHIP

The following have been awarded Honorary Fellowship of the Society:—

Lord Brabazon of Tara, M.C., Fellow of the Society, Past-President.
Sir Frederick Handley Page, C.B.E., Fellow of the Society, Past-President.
M. Louis Breguet, President, A.F.I.T.A.

## MEDALS AND AWARDS

The following medals, awarded by the Council, were presented at The Thirty-Seventh Wilbur Wright Memorial Lecture on the 28th April 1949:—

*British Gold Medal for Aeronautics*

Mr. S. Camm for his outstanding work in the design and development of fighter aircraft.

*British Silver Medal for Aeronautics*

Lt. Commander (A) E. M. Brown, R.N., for his outstanding achievements in advancing the technique of deck landing.

*Society's Bronze Medal*

Captain R. N. Liptrot for his work on helicopters, and Mr. E. N. Twining for his pioneer work in the design of model aeroplanes.

*Wakefield Gold Medal*

Mr. E. S. Calvert for his work on the development of airport lighting.

*George Taylor (of Australia) Gold Medal*

Mr. G. R. Edwards for his paper on "Problems in the Development of a New Aeroplane."

*Edward Busk Memorial Prize*

Mr. W. Stewart for his paper on the "Flight Testing of Helicopters."

*Herbert Ackroyd Stuart Memorial Prize*

Mr. W. H. Lindsey for his paper on "Development of the Armstrong Siddeley Mamba Engine."

*Usborne Prize*

Mr. D. R. Maguire for his paper on "Enemy Jet History."

*R. P. Alston Memorial Prize*

Mr. B. A. G. Mcgowan for his work in the flight testing of gliders.

## JOURNAL PREMIUM AWARDS

The following Premium Awards have been made for papers published in the JOURNAL during 1948:—

B. Thwaites, The Production of Lift Independently of Incidence, Ten Pounds, February 1948 Journal.

H. G. Conway, M.A., F.R.Ae.S., The Kinematics of Undercarriage Retraction, Five Pounds, February 1948 Journal.

E. G. Bowen, M.Sc., Ph.D., O.B.E., Delays in the Flow of Air Traffic, Five Pounds, April 1948 Journal.

T. Pearcey, A.R.C.S., Delays in the Flow of Air Traffic and Delays in Landing of Air Traffic, Ten Pounds, April and December 1948 Journals.

Professor G. T. R. Hill, M.Sc., F.R.Ae.S., The Nature of the Distortion of Swept-Back Wings, Five Pounds, March 1948 Journal.

A. W. Morley, Ph.D., M.Sc., A.F.R.Ae.S., Equilibrium Running of the Simple Jet-Turbine Engine, Five Pounds, May 1948 Journal.
H. L. Cox, M.A., F.R.Ae.S., and E. Pribram, D.E., A.F.R.Ae.S., The Elements of the Buckling of Curved Plates, Ten Pounds each, September 1948 Journal.
D. J. Lyons, B.Sc., A.F.R.Ae.S., Some Problems of the Stability and Control of Large Aircraft, Fifteen Pounds, November 1948 Journal.
A. Robinson, M.Sc., A.F.R.Ae.S., Rotary Derivatives of a Delta Wing at Supersonic Speeds, Ten Pounds, November 1948 Journal.
J. Norman, A.F.R.Ae.S., and S. F. Wilkinson, A.R.Ae.S., Aircraft Servicing, Five Pounds each, November 1948 Journal.

## THE AERONAUTICAL QUARTERLY

The Council have great pleasure in announcing the publication of *The Aeronautical Quarterly*.

*The Aeronautical Quarterly* enables papers describing new and original work, or papers reviewing the progress in some specialised field of activity, to be published. Through the medium of its pages results of these researches will be made available to all workers or design groups concerned with aviation.

To ensure that the high standard set in *The Aeronautical Quarterly* shall be maintained, the Council have been fortunate in the assistance which has been given to them by an Editorial Board and many of the leading authorities in various branches of aeronautical engineering and science who have willingly offered their co-operation to act as referees on all papers submitted for publication.

The Council of the Society was greatly encouraged to sponsor this new venture by the response it received from members of the Society to its enquiry as to the need of such a publication. The future of *The Aeronautical Quarterly* is dependent on the support it receives; it is hoped, therefore, this support will be forthcoming from all sections of the Society, so that the success of *The Aeronautical Quarterly* is assured.

The price of each issue of *The Aeronautical Quarterly* will be 10/3 post paid to non-members and 7/9 to members.

The contents of the first number are:—

| | |
|---|---|
| Flutter of Systems with Many Freedoms .. .. .. | W. J. Duncan |
| Control Reversal Effects on Swept-back Wings .. .. | H. Templeton |
| Estimation of the Effects of a Parameter Change on the Roots of Stability Equations .. .. .. .. | K. Mitchell |
| Note on Propeller-Turbine Reduction Methods .. .. | E. C. Pike |
| Determination of the Drag of Jet-Propelled Aircraft in Flight | G. W. Trevelyan and D. R. Blundell |
| Notes on the Linear Theory of Incompressible Flow Round Symmetrical Swept-back Wings at Zero Lift .. .. | F. Ursell |
| Calculation of Downwash Behind a Supersonic Wing .. | G. N. Ward |

## GRADUATES' AND STUDENTS' SECTION—1949-50 COMMITTEE

The following were elected to the Section Committee for the 1949-50 Session at the Annual General Meeting held on 23rd March 1949:—

*Chairman*: J. W. F. Housego (Handley Page Ltd.); *Vice-Chairman*: P. T. Fink (Imperial College); *Committee Members*: M. K. Bowden (Northampton Engineering College), W. M. Brady (Vickers-Armstrongs Ltd.), J. C. Gibbings (Queen Mary College), F. G. Irving (Imperial College), J. R. Miller (Hawker Aircraft Ltd.), J. K. Pudney (Fairey Aviation Co. Ltd.), J. G. Roxburgh (Handley Page (Reading) Ltd.), A. Stanbrook (Fairey Aviation Co. Ltd.); *Hon. Secretary*: M. C. Campion (Vickers-Armstrongs Ltd.), 6a Mildenhall Road, Clapton, E.5; *Assistant Hon. Secretary*: D. A. Thurgood (Vickers-Armstrongs Ltd.), Sunnyside Bungalow, Portsmouth Road, Ripley, Surrey.

## THE ELLIOTT MEMORIAL PRIZE

The Elliott Memorial Prize has been awarded to 582892 Cherry, R. O. G., Fitter IIE, Leading Aircraft Apprentice, the Aircraft Apprentice of the May 1946 Entry who obtained the highest marks in the General Studies Examination.

The award will be presented at the Passing Out Prize Giving at the R.A.F. station, Halton, on 1st June 1949.

## THE BUSK STUDENTSHIP IN AERONAUTICS

There is a vacancy for the Busk Studentship in Aeronautics for 1949. Application forms may be obtained from the Professor of Aeronautical Engineering, Engineering Laboratory, Cambridge, to whom they must be returned by the 1st June 1949.

This Studentship was established in memory of Edward Teshmaker Busk who lost his life in 1914 while flying an experimental aeroplane. The Studentship is of the value of about £165, tenable for one year from the 1st October 1949. It is open to any man or woman who is a British subject and of British descent who has not attained the age of 25 years on 1st October 1949. The object of the Studentship is to enable the holder to engage in research in aeronautics of the type in which Edward Busk was specially interested, that is, of the type involving experimental as well as mathematical investigation.

## GRADUATES' AND STUDENTS' SECTION

**Thursday, 12th May 1949**—Pressure Cabins, by D. Cardwell, B.Sc., A.C.G.I., A.F.R.Ae.S., Royal Aircraft Establishment.

In the Library of the Royal Aeronautical Society, 4 Hamilton Place, W.1, at 7.30 p.m. *The Library will not be open before 7 p.m.* Visitors are welcome. (No tickets are required, but the attendance sheet must be signed.)

*Visits*

A visit to Croydon Airport will be made on Saturday, 18th June 1949. Members wishing to take part in this visit should write to the Assistant Hon. Secretary, Mr. D. A. Thurgood, Sunnyside Bungalow, Portsmouth Road, Ripley, Surrey, not later than 1st June 1949.

## BRANCH NOTICES

### BROUGH BRANCH

The Second Annual Flying Display organised by the Brough Branch will be held at Brough Aerodrome (kindly lent by Blackburn Aircraft Ltd.) on Saturday, 18th June 1949. Proceeds will be in aid of the R.A.F. Benevolent Fund and the committee are arranging a full programme of events which will again include the £100 Cirrus Trophy Competition for Cirrus-engined Auster aircraft. It is hoped that Branch officials and others will make a note of the date and if possible respond to the invitations which will be forwarded to them in due course.

The display will begin at 2 p.m. Gates open at 11 a.m.

Arrival time for visiting aircraft from 11 a.m.

Admission to public enclosure 2/6, cars and motor cycles 1/-.

Further information may be obtained from the Secretary: F. A. Wilkinson, A.F.R.Ae.S., Design Office, Blackburn Aircraft Ltd., Brough, E. Yorks.

### WEYBRIDGE BRANCH

*Lectures*

**Wednesday, 18th May 1949**—Junior Prize Lecture, by Branch members.
**Wednesday, 1st June 1949**—General Meeting.

Meetings will be held at Vickers-Armstrongs Ltd., Weybridge Works, at 6 p.m., unless previous notice is given.

*Visits*

The following visits are being arranged for which further particulars may be obtained from the Hon. Secretary, J. H. Sinclair, A.R.Ae.S., Vickers-Armstrongs Ltd., Weybridge, Surrey:—

**21st and 28th May 1949**—Tumblin Bay, Decompression Chamber, Wind Tunnels, Mechanical Test Department, Process Department, Erecting Shop, Drawing Office, Wisley Aerodrome.

**11th June 1949**—National Physical Laboratory, Teddington.

— **June 1949** (date to be arranged)—Tilbury Docks to view the new Orient liner *Orcades*.

— **July 1949** (date to be arranged)—Bristol Aeroplane Co. Ltd. to view the Brabazon.

*Note.*—These visits are restricted to members of The Weybridge Branch.

## JOURNAL BINDING

The new prices of binding of Journals will be as follows:—

| | | |
|---|---|---|
| 1948 Volume .. .. | 14s. 6d. | (including packing and postage) |
| Previous Volumes .. | 16s. 0d. | ,, ,, ,, ,, |
| Cases for 1948 Volume | 6s. 0d. | ,, ,, ,, ,, |

Journals should be sent direct to the Lewes Press, Friars Walk, Lewes, Sussex, and the remittance to the Secretary at the Offices of the Society.

Requests for cases, with remittances, should be sent to the Secretary at the Offices of the Society.

## CHANGES OF ADDRESS

To assist in keeping the records of members correct and up to date the Secretary will be glad if all members will notify him as soon as possible of changes of address.

When notifying changes please give the following particulars:—

Name (in block letters).
Grade of membership.
New address (in block letters).
Old address.

Changes of address must be received *before the 15th of the month* in order to be effective for the Journal for the following month.

## NEW MEMBERS

The following new members have been elected recently:—

*Associate Fellows*

Arthur William Albert Andrews, Frederick Duncan Atkinson, Geoffrey Townsend Dobson (from Graduate), Marc Peter Le Lohe, Henry John Louch.

*Associate*

Wreford Hilton French.

*Graduates*

William Norman Bainbridge (from Student), Harold Caplan (from Student), Peter Alex Marks (from Student), Denis Roland Murrin (from Student), George Robert Prime (from Student), John Edward Rapson (from Student).

*Students*

Manna Lal Agarwala, Clifford John Benjamin, Lyndon Spencer Compton, Laurence Ivan Cook, Jack Samuel Doyle, Jonathan Huntley Wallace Shannon, William Victor Williams.

## ACKNOWLEDGMENTS

The Council acknowledge with grateful thanks the following gifts: A framed picture of early balloons, which is of great historical interest, from the Air Registration Board; A copy of " Flygmotorer " from G. Gudmundson, Associate; Periodicals and air guides from Mrs. Walker Sinclair, Companion; Periodicals from Major B. W. Shilson, Fellow; and back numbers of the Journal from J. W. Loader, Associate Fellow.

## ADDITIONS TO THE LIBRARY

Pamphlets in italics with location reference following in brackets. Books marked * or ** may not be taken out on loan.

A.a.356—Yacht Racing, the Aerodynamics of Sails. Manfred Curry. Scribners. 1948.

BB.g.2—Quality Control. N. L. Enrick. Industrial Press, N.Y. 1948.

E.a.21—Flygmotorer. G. Gudmundson. Hermods Korrespondensinstitut. 1949.

G.e.E.69(F)—Les conceptions Americaines relatives à la trempabilité de l'acier. Yves Dardel. Centre du Documentation Siderurigique. 1948.

L.k.68—Aviation Medicine in its Preventative Aspects. J. F. Fulton. Oxford University Press. 1948.

M.c.83—Principles and Practice of Radar. H. E. Penrose. Newnes. 1949.

*X.c.B.35—Dictionary of Guided Missile Terms. Research and Development Board, National Military Establishment, U.S.A. 1949.

### A.R.C. Reports and Memoranda

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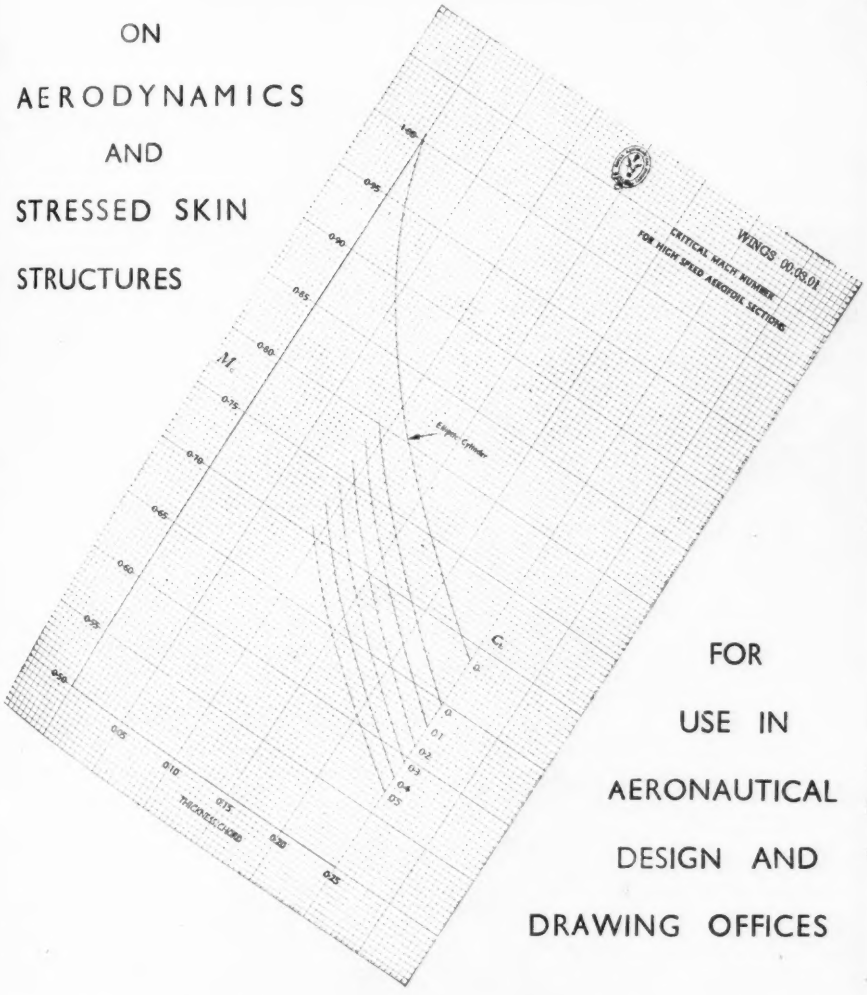


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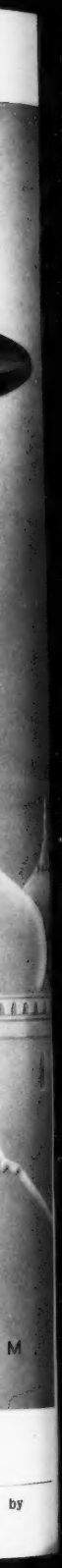
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